Learning Under Multiple Public Information Sets*

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Abstract

The paper analyses how individuals form expectations when they are uncertain about the current state of the economy and must decide their set of relevant information. Using evidence from a period with multiple inflation statistics in Argentina (2007-2011), I analyse how individuals use price information from their own consumption basket in order to choose which public data to use in forming inflation expectations. I characterize the consumption-basket inflation rates' distribution by using online prices from one of the leading retailers of the country combined with expenditure information for roughly 25,000 households. Under uncertainty about the current aggregate level of prices, households' inflation expectations diverge in line with the change in prices from the goods they purchase. To disentangle the effect of information uncertainty from the increase in relative price dispersion, I model inflation expectations through the eyes of a Bayesian learner that uses public and idiosyncratic information but knows signals may be potentially biased. Results suggest that idiosyncratic information on prices may affect economic decisions and outcomes, even in situations with a unique inflation statistic where there are doubts about its quality.

JEL Codes: E31, D83, D84.

Keywords: Bayesian learning, information sets, inflation perceptions and expectations, misleading information.

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I Introduction

How do individuals form expectations about the future when they are uncertain about the current state of the economy? In particular, how do they process available information in situations in which they have imperfect knowledge of prior endogenous variables, and face signals from multiple public information sets?

Informational frictions may induce individuals to form perceptions about the current state of the economy before making economics decisions. Perceptions, in turn, can have an effect on what individuals expect and on their current and forward-looking decisions. The role of public statistics is then crucial, they shed light on current economic conditions[1] which are not directly observed by individuals. What is more, economic indicators may act as a coordination device for individuals' expectations (Cornand, 2006), helping to avoid the heterogeneity caused by the misinterpretation of such conditions. Examples of these kind of variables (and indicators), usually released by government agencies, much more informed than individuals, include GDP, consumer price indexes, real exchange and unemployment rates.

Quite often, and probably due to the importance of the information contained in the statistics, other organisms, such as international organizations and private agencies, release alternatives measures of the same indicators. Differences in public statistics may emerge as a result of measurement errors or sampling variation[2], but also as a result of misleading information, released with the aim of modifying individuals' perceptions and expectations by taking advantage of the asymmetrical information. What the individual believes to be the nature of the difference between public statistics may determine their choice among alternative information sets.

In this paper, I focus on the role of consumer price indexes (CPIs) on inflation expectations. I use recent evidence from a period in Argentina characterized by the existence of multiple consumer price indexes. The example is particularly interesting because prior to this period there was virtually only one official CPI widely accepted as the indicator for the general price level. However, at the beginning of 2007 an institutional change in the government agency in charge of releasing the index generated doubts in some people who began to believe that the

[1] Bureau of Economic Analysis (2018)

[2] e.g., import price indexes-based real exchange rates *vs.* CPI-based real exchange rates

information was manipulated. As a response a group of alternative price indexes emerged, both from official and unofficial sources. After a few months, individuals were confronted with multiple series of highly dispersed inflation statistics.

A natural consequence of the different ways in which people could have processed the available information on the evolution of aggregate prices is the cross-sectional heterogeneity in inflation expectations. Even when individuals' learning behavior is similar and when initial priors about inflation are homogeneous, individuals' expectations may diverge as a response to the weighting of public indicators and their own idiosyncratic information on prices; the one that comes from their experience as a consumer.

While average expected inflation closely followed unofficial CPIs statistics, the mere presence of these indicators could not explain the observed heterogeneity in inflation expectations in Argentina's case. The paper shows that, under uncertainty about the current aggregate level of prices, households use their own consumption basket pricing information both to decide their set of relevant information and to form inflation expectations. The heterogeneous behavior observed in expectations is consistent with consumption-basket inflation rates throughout the period between 2007-2011: households with a lower level of education experienced higher average rates of consumption-basket inflation and had higher expected inflation rates than households with a higher level of education[3].

The paper contributes to the literature that attempts to understand how expectations regarding inflation are formed. I help to explain heterogeneous inflation expectations, one fact highly observed in the data, as a consequence of the different price experiences of subgroups of households, complementing the findings of previous studies that established the importance of the frequency with which the information sets are updated (Madeira & Zafar, 2015), the way expectations are formed (Evans et al., 2001), and the weights assigned to past realizations of endogenous variables from a single information set (Malmendier & Nagel, 2016). In particular, the article is linked to the works of Jonung (1981) and Cavallo et al. (2016, 2017), which highlight the importance of inflation perceptions on expectations. A fact also documented by Bryan and Venkatu (2001). However, while previous research uses survey-based measures of inflation perceptions, I show the importance of the actual consumption-basket inflation rate experienced by each

[3] The relationship holds for almost the entire period but for some moths in 2009.

household on both, perceptions and expectations[4]. Johannsen (2014) shows that demographic groups with a more dispersed consumption-basket inflation rates' distribution also have a greater disagreement about inflation expectations, I focus here on the mean response to study the expectation formation process under uncertainty.

In measuring "inflation" at the household level, and due to the lack of *a priori* reliable information on prices, from both official and unofficial sources, I combine web scraping information on daily prices from an online retailer with household expenditure information from the national consumer survey. In particular, I construct about 25,000 Laspeyres fixed-base quantity price indexes at the household level by weighting category-level price indexes (developed by Cavallo, 2013) with expenditure weights for each household and product category. The empirical approach is related to the studies that have measured household-specific rates of inflation (e.g. Michael, 1979; Hagemann, 1982; Johannsen, 2014; Coibion, Gorodnichenko, & Hong, 2015; Kaplan & Schulhofer-Wohl, 2017), but use web scraping data on prices rather than sub-indexes of the CPI or scanner data. In line with previous results, I find an important cross-sectional heterogeneity.

The article complements the discussion initiated by Cavallo et al. (2016) on how individuals learn from manipulated statistics. Using experimental data from the same period in Argentina, they show that individuals were neither naive users nor complete negators of official statistics but rather used their knowledge of the bias induced by the government to extract the relevant information. Their findings can be interpreted as an equilibrium in the learning process as a result of two assumptions: the existence of an observable unbiased measure of aggregate inflation and a fixed bias over time in the official economic indicator[5]. My paper contribute to this discussion by avoiding to take a position on the quality of economic indicators, by analysing the effect of the uncertainty created by the

[4]Nevertheless, it should be noted that this is not a paper on inflation perceptions formation—other determinants, like cognitive limitations and rational inattention, are not explicitly considered here (e.g. Cavallo et al., 2017)—, but rather on the effect of the true experience of each individual as a consumer in their inflation expectations.

[5]They proposed a Bayesian learning model with biased statistics to explain the way in which individuals recognize the bias in official statistics. These findings may be somewhat limited by the fact that the empirical evidence was obtained in December 2012, six years after the potential manipulation of statistics began. Experimental results shows that individuals were acting as if the bias in the official measure were 10%. Although this result might be ex-post accurate (see Figure (1)), does not necessarily reflect the evolution of individual perceptions and expectations.

existence of multiple sources of information, all of which may face unpredictable and continuous changes, and by recognizing that the idiosyncratic information on prices, from the consumption basket, may be the only source of information of which the individual knows its quality[6].

To explain individual behavior, the paper develops a Bayesian learning model of inflation expectations. The model allows to map multiple public and idiosyncratic information sets into expectations. As in Cavallo et al. (2016) the key feature of the model is to introduce a potentially biased public statistic of the past rate of inflation. I extend their model by introducing idiosyncratic information (i.e., private information only available to each household) and by studying environments in which none public signal are unbiased.

The rest of the paper is organized as follows. Section (II) analyses the evidence from Argentina's episode of multiple inflation statistics. Section (III) introduces the Bayesian learning model that explains how individuals can form perceptions and expectations about aggregate inflation by using price information of their own consumption basket. Section (IV) discusses why a government may have incentives to release biased statistics if households use all sources of information available to them. Section (V) extends the baseline specification to allow for sources of unbiased information. Section (VI) discusses further extensions to the empirical analysis and Section (VII) offers concluding remarks.

II Days of multiple inflation statistics

In January 2007, the Argentinian Government decided to apply what some international and national institutions, media and academics spheres believed was a *de facto* downward intervention on the official Consumer Price Index (CPI). The government replaced the main authorities of the INDEC, the decentralized government agency in charge of releasing the official CPI, in an unusual and controversial way. Although there is no broad consensus as to the reasons for this decision, three factors stand out among those who believed the consumer price index was manipulated: (a) to pay lower interest rates on inflation-linked bonds,

[6] Cavallo et al. (2016) argues that we can identify the unofficial statistics as a measure of individuals' memories of the prices from the goods they have purchased. Although this assumption is sensible to capture up to some degree the experimental results of the paper, it does not explain the appearance of multiple disperse measurements and the heterogeneity in expectations.

(b) to lower inflation expectations, and (c) to allow the government to claim that inflation was under control in order to enhance public opinion in anticipation of the presidential election of October 2007[7]. These new authorities released the monthly CPI index from January 2007 to December 2015 with some interruptions and one mayor methodological change[8].

How reliable these official statistics were began to be a public concern almost immediately. In mid-February 2007, the opposition in parliament asked for an investigation and the issue appeared broadly covered in the media. As a response to public concerns, private consultancy groups, universities, and the offices of statistics of some provinces that were not aligned with the federal government engaged in generating and delivering alternative measures, and thus a heterogeneous group of new price indexes progressively emerged.

At the same time, according to most unofficial sources, the economy was beginning experience an increase in the level and volatility of inflation in comparison to the immediate previous years[9]. What's more, whenever households assigned a positive value to unofficial signals, they ended up validating (in the form of perceived inflation) the change in the inflation pattern. Moreover, any consent that the information had some degree of validity implicitly increased the opportunity cost of ignoring inflation.

Figure (1) shows the behavior of inflation according to the main official statistic, an unofficial statistic[10] and inflation expectations for the period between 2007 and 2013. The behavior of inflation expectations suggests that, on average, households assigned a positive value to unofficial inflation statistics. The forecast errors reflected in the difference between inflation expectations and unofficial figures is consistently lower than that reflected in the difference between expectations and the official measure[11].

[7] Although the annual inflation was already high at the end of 2006, it was still below two digits. Due to the history of high inflation in the country, the inflation level is always an important concern for voters. It seems that the two-digit inflation operates as a threshold for media and public opinion when it comes time to select a candidate (La Nación, 2007)

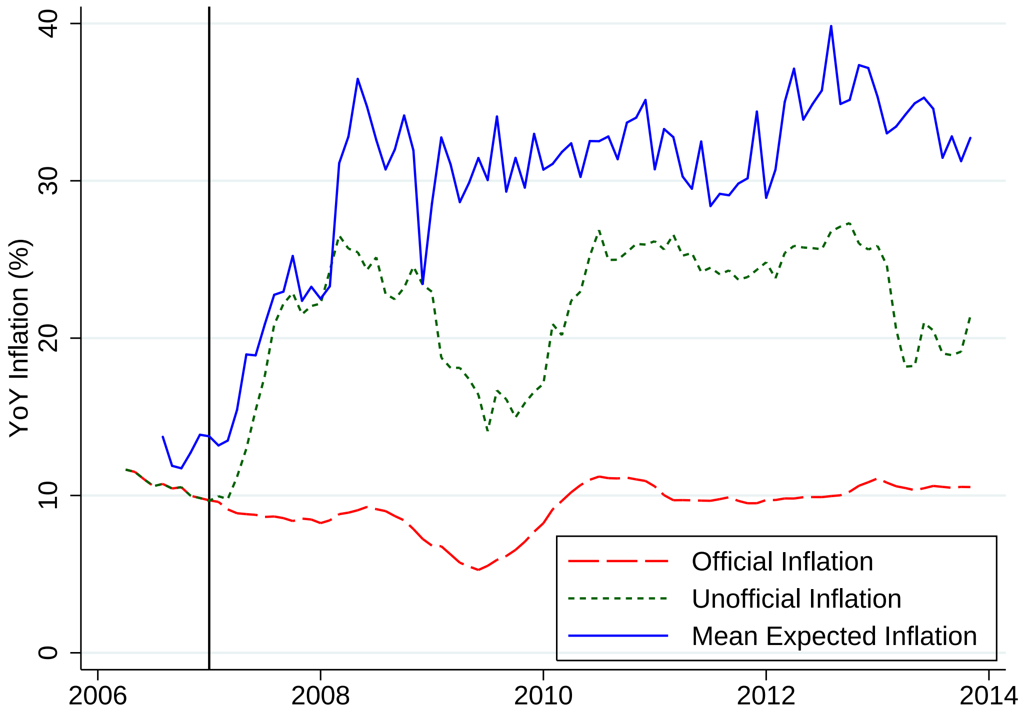
[8] See Cavallo et al. (2016) for a detailed timeline of the events regarding the official inflation statistics in Argentina for the respective period.

[9] See Drenik and Perez (2018) for a discussion about the general macroeconomic conditions at the time and the change in the inflation path

[10] I restrict attention here to the unofficial measure computed by Price Stats. See Appendix A for details

[11] Appendix A shows the time series of forecast errors.

Figure 1: Official, Unofficial and Expected Argentinian Inflation, 2007-2013



Note: Expected Inflation is from the Encuesta de Expectativas de Inflación (UTDT). The official CPI is IPC INDEC (2008=100) and the unofficial CPI is the CPI computed by Price Stats (Inflación Verdadera)

However, how individuals chose (if, in fact, they did choose) among alternative inflation statistics to form perceptions about the aggregate level of prices and whether or not these perceptions affect expectations is still an interesting open question. First, the party in government retained power during this period: the former president's wife won the presidential election in October 2007 and was reelected in October 2011. Since she won more than 45% and 54% of the votes, respectively, voters showed considerable support for the administration, thus ruling out an immediate loss of credibility resulting from this episode, at least one that is extendable to all areas of the government. Second, the dispersion among alternative indexes was very important; there was even at least one unofficial measure tracking official inflation closely[12]. Third, although some (non-conclusive) evidence indicated the possible manipulation of statistics (e.g. Cavallo, 2013)

[12] Cavallo et al. (2016) for a detailed list of unofficial inflation indexes

there was never a clear consensus in the procedure that was carried out[13] nor has it been clear that this evidence was public knowledge. Furthermore, even the judicial authorities in April 2018, under a new administration with different political leadings, dismissed the case against the INDEC authorities accusing them of manipulating statistics, arguing that there was no evidence to prove fraudulent handling[14]. Finally, as Drenik and Perez (2018) points out, there was no consensus regarding the actual validity of alternative measurements.

In summary, although the behavior of the median household was consistent with the evolution of most of the unofficial statistics, the question remains as to how they identified (if possible) the unbiased statistics, necessary to extract the relevant information, is unsolved. This question is even more relevant in a situation such as the one described here, in which there were not one but several unofficial statistics with implicitly heterogeneous inflation rates and time series dynamics.

It is natural to expect that this uncertainty generates heterogeneous responses in households expectations. The distribution of expectations reflects, at least to some extent, the way in which households process available public and idiosyncratic information[15], in other words, the way they perceive current inflation. Figure (2) shows annual inflation expectations concerning the aggregate level of prices grouped by individuals' level of education[16]. As we can observe from the behavior of expectations during this period, individuals who had no more than high school education expected on average higher inflation than individuals with a higher level of education. In fact individuals with a lower level of education were 85% more likely to expect higher inflation[17].

[13]Some argue that authorities manipulated statistics by misreporting the prices with the highest increase in each period, other argue that they changed the weight of those prices in the reference basket. Retrospective observation allows us to suspect that the official CPI was manipulated to fluctuate around an annual inflation rate of 10 percent, at least working as an upper limit. However, it was not easy to find a pattern in monthly data for the dates on which the inflation rates were released

[14]See La Nación (2018). It should be mentioned that the appeal process is still open in December 2018

[15]Another possibility would be that they form expectations differently. However, in this work I assume homogeneity in the process of expectations formation.

[16]A similar, but weaker, pattern is observed by grouping individuals by regions. The city of Buenos Aires faced on average less inflation than the rest of the country.

[17]Only in 6 months out of 42, households with higher level of education expected more inflation than households with a lower level of education

Figure 2: Mean Annual Expected Inflation by Level of Education, 2007-2011

Note: Expected Inflation is from the Encuesta de Expectativas de Inflación (UTDT).

The next subsection provides evidence on households' price information based on their own consumption basket. I then show its link with the heterogeneity observed in expectations.

i Households' idiosyncratic information on prices

Each household faces an idiosyncratic evolution of prices based on their consumption basket. Under uncertainty about the evolution of aggregate prices, one way for households to infer which information set is the most accurate, that is, which is closer to the true data generating process, is by relying on their own idiosyncratic information. If that is the case, then inflation perceptions are determined, at least in part, by the actual consumption experience.

To identify household idiosyncratic information on prices that are free of potential source-bias during this period, I propose a new approach[18] to measure the evolution of prices at the household level. I use prices from an online retailer surveyed by the Billion Prices Project at MIT combined with household expenditure information from the national consumer survey[19]. Scrape data on prices and household expenditures are organized in product categories according to the Bureau of Labor Statistics. I use Cavallo (2013) category-level price indexes for the period between October 2007 and March 2011[20]. 53 category-level indexes are constructed by first computing the daily unweighted geometric mean of price changes within each product category[21]. Then, I construct expenditure weights for each household and for each product category by using information from the national consumer survey. The proportion of total household expenditure in each category represents its weight in the respective consumption basket. Finally, I compute the weighted arithmetic mean of all category-level price indexes for each household[22].

25, 833 Laspeyres fixed-base quantity price indexes are calculated through 1, 265 days. The average across households of the mean annual daily inflation is 20.23% with a standard deviation of 2.54, while the average cumulative inflation for the entire period is 97.86% with a standard deviation of 20.04[23]. Interestingly, if we group households by their level of education, the average mean annual daily consumption-basket inflation for the ones with a lower level of education, those which at most hold a high school certificate, is 20.45%, 1.12 percentage points higher than the average across households with a higher level of education, those who have at least incomplete tertiary studies. The cumulative consumption-basket inflation experienced through the period are 98% and 91%, respectively[24].

Figure (3) shows the daily average household mean annual consumption-basket inflation by according to the level of education from October-2007 to March-2011.

[18] See Michael (1979), Hagemann (1982) and Johannsen (2014) for an overview of the traditional approach to characterize the inflation distribution

[19] Appendix A summarizes information on the data sets.

[20] The data set is only publicly available for this period, see Appendix A. I use the code from the paper in computing these indexes.

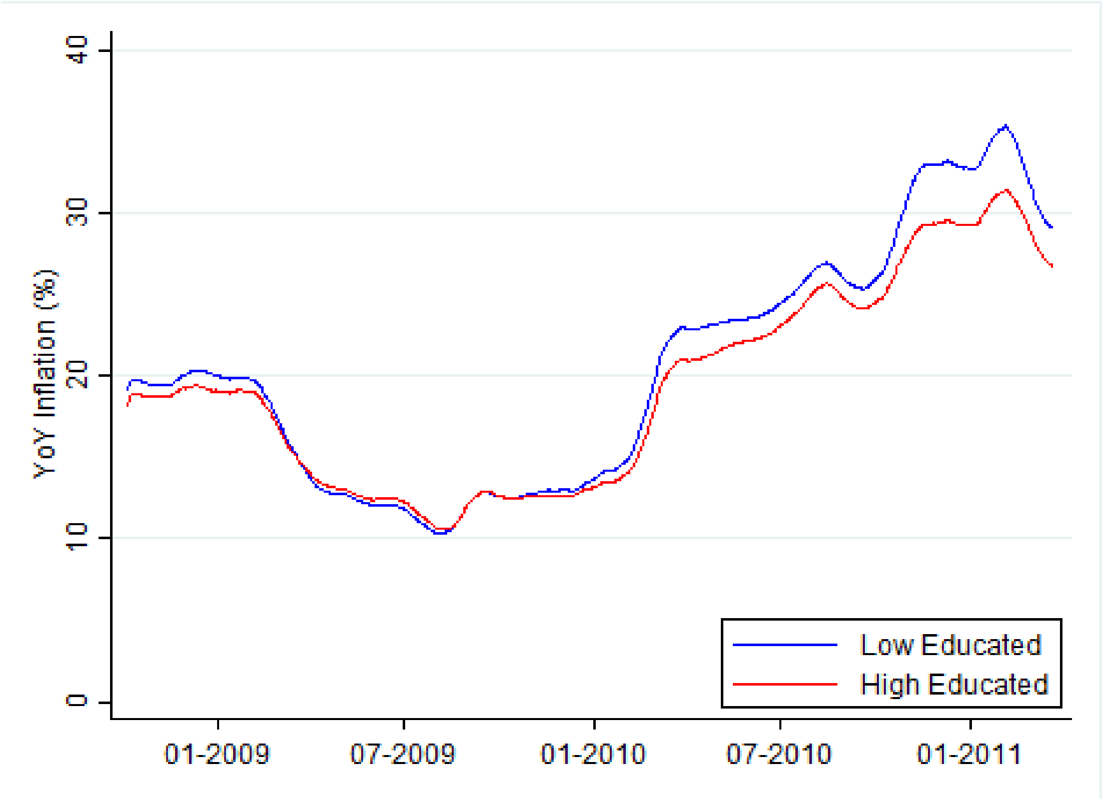
[21] See Appendix A and Cavallo (2013) for a detailed explanation of the methodology.

[22] See Appendix A for details.

[23] No significant differences are found if we consider the interquartile range. See Appendix A

[24] Although the standard deviation of both measures are higher than the difference between means, two sample t-tests show that the differences between means are statistically significant at 1%. See Appendix A for additional information on both groups

Figure 3: Mean Annual Household Inflation by Education, 2007-2011



Note: The mean annual household inflation is the daily average of households' annual inflation rates grouped by level of education. Each household inflation rate is measure by using prices from an online retailer combine with the expenditures weights surveyed in the Encuesta Nacional del Gasto de los Hogares 2004-2005.

Households with a lower level of education faced more inflation than households with a higher level of education in almost the entire period. The main reason is that the households with a lower level of education are in general at the bottom of the income distribution[25] and have a consumption basket with a higher proportion of food which experienced a faster increase in prices[26]. Except during some part of 2009, when the country was facing a recession and prices were slightly decreasing, the average consumption basket of households with a lower level of education became more expensive relative to the one of households with a higher level of education.

Substitution effect. One valid concern is how well the Laspeyres fixed-base

[25]ADD EVIDENCE from ENGH04-05

[26]See Table (3) in Appendix A

quantity price indexes can identify the evolution of prices of each group in a context of increasing inflation and a possible change in relative prices. Accordingly, it is necessary to differentiate between two substitution effects: the intra-category and the inter-category. The online prices data sets are better tools than standard CPIs in capturing the intra-category substitution effects. Discontinued products disappear automatically from the sample avoiding the bias created by imputed prices for temporary or permanent substitutions, usually used in CPIs (Cavallo, 2018). Furthermore, new products appear the same day they start being offered online. Unfortunately, the national consumption survey was not updated during this period to check variations in expenditure shares. A partial solution is to analyze more in deep the evolution of relative prices to identify potential sources of systematic bias between both groups.

To investigate in which way relative prices diverge, I construct two group-specific CPIs that track the evolution of prices of both type of households, lower and higher educated. I combine the average weights expenditures of each type with prices from the online retailer[27] In March-2011 the CPI of the group of households with the lower level of education reaches 199 points, 4.3% more than the CPI of the group of households with the higher level of education. Table (1) reports the breakdown of the relative discrepancy between group-specific CPIs (for some selected categories) by decomposing the contribution of the increase in prices of each category-level index with respect to the total average increase in prices, from the contribution of the relative importance of each category in its respective CPI[28].

As can be seen from the table, the Uncooked Beef Steaks category explains 2.9 percentage points of the total discrepancy between both group-specific indexes. On the one hand, the price of beef steaks increases 67.1% more than the average increase in prices and the weight of this category in the lower educated CPI is 11.7%, 57.5% more than in the higher educated CPI. Some products contribute to the discrepancy for the opposite reason, its price growth at a lower rate than the average growth rate of prices and their weights in the higher educated CPI

[27] Note the difference with the previous exercise. Before, I identify a household-specific CPI and average the household-specific increase in prices by their level of education. Now, I identify the representative weights of each type, combine with online prices and compare the evolution of both group-specific CPIs.

[28] Here I follow the approach in Hagemann (1982). See Appendix A for details

Table 1: Relative Consumption Basket-CPIs Discrepancy by Level of Education

Category Name	Index	Mean LE	Mean HE	Price Change	Weights	Impact of Category
UNCOOKED BEEF STEAKS	319.6	11.7	7.4	0.6713	0.5750	2.8667
RICE	212.5	1.4	0.8	0.1113	0.7034	0.0639
BREAD	207.1	8	5	0.0828	0.5922	0.2459
PASTA	204.5	3.5	3	0.0693	0.1504	0.0315
CHICKEN	203.3	4.3	2.8	0.0631	0.5368	0.0946
OTHER FRESH FRUITS	161.3	3.1	3.8	-0.1568	-0.2005	0.1211
BOOKS	131.8	0.9	1.8	-0.3110	-0.4918	0.2790
HOME FURNITURE	128.6	1.5	2.1	-0.3274	-0.3139	0.2199
APPLIANCES	128.2	1.8	3.2	-0.3296	-0.4344	0.4622
TOTAL	191	100	100	0	-	4.3

Notes: the index column measures the average cumulative change in prices in each product category for the full sample. Mean LE and Mean HE show the weights of each product category in the their respective consumption basket. Price change captures the relative discrepancy between the cumulative change in prices in each product category with respect to the average cumulative inflation. Weights compares, in percentage terms, how different the low educated consumption basket-CPI weights are with respect to the reference basket. Impact of Category summarizes the contribution of each product category to the total discrepancy between both consumption basket-CPIs

are greater than in its lower educated counterpart. Books, Home Furniture and Appliances categories contribute in approximately 1 percentage point of the total discrepancy. This four categories of products plus Bread, Pasta and Chicken explain about 98% of the discrepancy between both CPIs at the end of the sample and represent a one-third share of the lower educated consumption basket[29].

Since the discrepancy between both group-specific CPIs is highly concentrated in a small subset of categories that represents a non-trivial expenditure share of the lower educated CPI, the above results rule out, at least to some extent, the possibility of a disproportionately substitution effect that occurs only in one of the consumption baskets, even when the proposed measures of consumption baskets may overestimate the actual inflation experience in both groups[30].

Cavallo et al. (2016) tests whether salient products with controlled prices affect inflation perceptions. They find no evidence in the short-run: even thought

[29] See Table (3) in Appendix A for the impact of the 53 categories

[30] Note that even if there is a substitution effect, the only way to close the gap between both group-specific inflation rates is if lower educated households substitutes products with higher inflation rates to products with lower inflation rates, closely to the rates of the higher educated households. Although this effect may be expected in the medium-run —there is no evidence of a demographic group that constantly faced higher inflation (e.g. Jonung, 1981; Michael, 1979)— it may not occur in the short-run. Habit formation (e.g. Fuhrer, 2000) may be one of the reasons among others.

controlled products have a substantially lower price increase than non-controlled ones, individuals have the same memory of price changes. One possible explanation is that individuals track their consumption-basket inflation rate, rather than the evolution of individual price changes, and then assume a similar variation for all products. Results in this paper also suggest that public statistics play a role in coordinating perceptions. Unfortunately, the BPP dataset is not publicly available for the period with price controls to test these hypothesis.

ii From idiosyncratic information to expectations

Even if we accept the possibility that people's perceptions about current inflation are affected by their idiosyncratic information on prices and, hence, differ across households, perceptions may have no significant effect on shaping inflation expectations. They might be explained by other additional factors such as professional forecasts, media comments, expectations about the evolution of the monetary policy and the exchange rates, simple extrapolation of public statistics, etc.

To assess whether idiosyncratic information on prices affects households' expectations and explains part of the heterogeneity observed in the data, I conduct pseudo-group regressions by classifying individuals according to their level of education and by using the corresponding average of the measures of consumption-basket inflation rates as the independent variable. Table (2) shows the responsiveness, by level of education, of the average household inflation expectation to the average price movements of their own consumption basket. Results indicate a positive and statistically significant effect of the consumption-basket inflation experienced by each group on their own inflation expectations.

More interestingly is that the estimated effect of consumption-basket inflation on expectations is similar for both groups, roughly one-half a percentage point. This means that for each percentage point increase in consumption-basket inflation, there is an increase of 50 basis points on inflation expectations. Given that the consumption-basket inflation rate of lower educated households is higher than the one of higher educated ones, on average part of the difference in expectations between both groups is explained by their consumption-basket inflation experienced.

The difference in expectations is more important than the difference observed

Table 2: Average Response by Level of Education of Household's Inflation Expectations to Consumption Basket Price Movements

	(1) $\pi^{e,H}_{t+12\|t}$	(2) $\pi^{e,H}_{t+12\|t}$	(3) $\pi^{e,H}_{t+12\|t}$	(4) $\pi^{e,L}_{t+12\|t}$	(5) $\pi^{e,L}_{t+12\|t}$	(6) $\pi^{e,L}_{t+12\|t}$
π^{O}_{t-1}	-0.640**	-1.515**	-1.392**	-0.773*	-1.067	-1.117
	(0.304)	(0.652)	(0.617)	(0.401)	(1.053)	(1.060)
π^{U}_{t-1}	0.958***	0.230	0.158	1.138***	0.0485	0.0431
	(0.0809)	(0.237)	(0.249)	(0.0989)	(0.384)	(0.392)
$\pi^{CB_H}_{t-1}$		0.548***	0.570***			
		(0.138)	(0.140)			
$\pi^{CB_H}_{t-2}$		-0.0537	-0.0706			
		(0.151)	(0.137)			
$\pi^{e,H}_{t+11\|t-1}$			-0.166			
			(0.137)			
$\pi^{CB_L}_{t-1}$					0.588*	0.611*
					(0.303)	(0.299)
$\pi^{CB_L}_{t-2}$					-0.0192	-0.000935
					(0.299)	(0.318)
$\pi^{e,L}_{t+11\|t-1}$						-0.107
						(0.179)
Constant	14.18***	21.09***	26.63***	13.48***	22.16***	24.79***
	(2.581)	(3.923)	(5.355)	(3.709)	(7.178)	(7.300)
Trend	NO	YES	YES	NO	YES	YES
Observations	90	29	29	90	29	29

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Column 2-3 display the results obtained from pseudo-group regressions of the average inflation expectations of households with higher level of education on the mean inflation rate of their own consumption basket. Column 1 displays the responsiveness of household inflation expectations on the inflation rate of the official and unofficial indicators only.

in the consumption-basket inflation rates—Figures (2) and (3)—. One possible explanation is that individuals assessed the inflation they experienced in their consumption baskets in terms of their real purchasing power. Households with lower levels of education were more likely to be employed in informal jobs than those with higher levels of education. This in turn implied that they participated to lesser extent in the yearly wage negotiations and therefore experienced a greater decrease in purchasing power than that reflected by the difference in consumption baskets nominal price increases.

Similarly, we cannot discard the possibility that survey expectations regarding the aggregate level of prices do not, in fact, reflect household beliefs about the aggregate level of prices but rather their beliefs about the evolution of their own consumption basket. This debate is a standard concern about the inflation expectations surveys (e.g. Armantier et al., 2013). In any case this argument does not break the link between idiosyncratic information on prices and expectations, but rather explain why they diverge.

A valid concern is whether lower educated households always have higher inflation expectations or whether they update their expectations more slowly, a documented regularity in contexts of low inflation (e.g. Madeira & Zafar, 2015). A rapid inspection of Figure (10) in Appendix A shows that these explanations are unlikely and should not represent an important part of this divergence, probably because rational inattention is to costly in an environment of moderately high inflation.

The fact that actual consumption-basket inflation rates affect households expectations is contrary to previous studies which have suggested that the evolution of prices experienced by each group is not able to explain the large heterogeneity of inflation expectations (Madeira & Zafar, 2015; Hobijn et al., 2009). The informational friction, originated in the absence of reliable information on the current state of inflation, and the environment of rising inflation force households to increasingly rely on their own price information, which comes from the products included in their usual consumption basket. Because the dispersion of relative prices increases with inflation (e.g. Alvarez et al., 2018), it pushes up the cross-sectional heterogeneity in consumption-basket inflation rates, which translates into a more disperse distribution of inflation expectations. These findings suggest that using information from the change in prices from the goods households purchase is not necessarily a cognitive limitation[31] but rather a "rational" response to the lack of reliable public information.

In the following section I propose a model to understand how, in a context with informational frictions and different public signals about the current state of aggregate prices, households form inflation expectations by using information on prices of products from their own consumption basket to weight the information coming from the public signals. Section (III) extends the work of Cavallo et al. (2016) by introducing heterogeneous households with idiosyncratic information on prices and by allowing the full set of signals, public and idiosyncratic, to be potentially bias.

III The model

To analyse the effect of information sets on inflation perceptions and expectations I propose a Bayesian learning model. The framework is capable of nesting multiple sources of information of the inflation rate. Some of these statistics are public, which means that all households observe them. Others are idiosyncratic in nature, i.e., only available to each particular household. Next subsections show how inflation dynamics and signals (statistics) interact.

i Inflation Dynamics

Household types only differ in their consumption experience and, hence, in their idiosyncratic information on prices. Household i consumes a consumption basket with an average price change given by

$$\pi^i \sim \mathcal{N}(\mu_i, \sigma_i^2) \tag{1}$$

Consumption-basket inflation may be different across households due to sampling variation (non-systematic bias) or due to the fact that households may systematically consume products (with different price changes) in different proportions

[31] Cavallo et al. (2017) notes the importance of cognitive limitations when individuals form inflation expectations. However, it is the use of price changes memories and not the actual price movements they experienced what determines their expectations. In other words, the inaccurate memory is what induces heterogeneity in expectations.

(systematic bias)[32]. The idiosyncratic variance reflects the dispersion of prices in each consumption basket type.

The inflation rate in the economy for any period t is the expenditure weighted sum of each consumption-basket inflation

$$\pi_t = \sum_{i=1}^{N} \alpha_i \pi^i \tag{2}$$

α_i is household i's share of total expenditure in the economy and N indicates total number of households. Hence, $\sum_{i=1}^{N} \alpha_i = 1$. Assuming $(\pi^1, ..., \pi^N)$ are mutually independent, and due to (1), the inflation rate[33] in the economy is normally distributed according to

$$\pi \sim \mathcal{N}(\mu,\, \sigma^2) \tag{3}$$

with $\mu \equiv \sum_{i=1}^{N} \alpha_i \mu^i$ and $\sigma^2 \equiv \sum_{i=1}^{N} \alpha_i^2 \sigma_i^2$.

Idiosyncratic Bias. In any given period t, the difference between the average price change in consumption basket i and the inflation rate can be the result of sampling variation or systematic bias in the idiosyncratic information, i.e., the bias coming from households with a consumption basket that substantially differs from the one that measures inflation. I define this difference as

$$b_t^i \equiv \pi_t^i - \pi_t \tag{4}$$

and its expected value is given by

$$E(b^i) = \mu_i - \mu \tag{5}$$

Equation (5) indicates how representative of the inflation rate is the evolution of household i's consumption basket prices; the higher is the bias of the idiosyncratic information, the less informative is their own consumption experience.

[32]It may also reflect the fact that different demographic groups pay systematically different prices for the same products. However, this is a dimension I cannot exploit in the data.

[33]Note that it is possible to express the inflation rate of a fixed market basket of goods and services as the weighted sum of households consumption-basket inflation rates as long as the fixed market basket is constructed with expenditure weights that reflect the share of total expenditure of each household. See Appendix B for further discussion

ii Idiosyncratic Signal

Distribution (1) determines household i's consumption-basket inflation (π_t^i) for every period t. Given b_t^i, π_t^i is an implicit signal of the evolution of the general level of prices. It can be written as

$$\bar{\pi}_t^i = \pi_t^i - b_t^i \tag{6}$$

Whenever $E(b^i) \neq 0$, the implicit signal is biased. It captures the idea that some households may consume from a subset of products with higher(lower) average price changes than the one measured by the representative consumption basket. In other words, $|E(b^i)|$ indicates how close (far) consumption-basket inflation of household i is from the true inflation rate.

iii Public Signals

Any agency in charge of releasing a measure of the general level of prices observes a subset of the total population of products. I assume that this random draw is unbiased[34] and normally distributed according to

$$\bar{\pi}^p \sim \Big(\mu,\, \sigma_p^2(\sigma^2)\Big)$$

The variance is a function of the true variance of the inflation dynamics and depends on how precise is the public measure p in measuring inflation. Households may not observe this measure directly but through a signal. Agency p releases

$$\pi^p = \bar{\pi}^p + b^p$$

which is observed by all households. Whenever $b^p \neq 0$, the signal is biased. Households cannot distinguish between the unbiased part of the signal and the bias.

[34] Note that in reality this measure may be biased due to an important number of different reasons, substitution bias, quality bias, new product bias and outlet bias, among others.

iv Inflation Expectations

Households' objective is to learn $E[\pi]$. In particular, households know the variance of the inflation dynamics in (3), but they have uncertainty about the mean value. I assume households form expectations for period $t+1$ conditional on the set of all relevant information sets available at period t. Because public information quality may not be good, households do not use directly the information they received to form inflation expectations. They know that public signals may be biased[35]. Additionally, households realize that their own information on prices may be a biased sample draw from the total population of products. Hence, in order to learn the underlying mean of the inflation dynamics, households need to learn the expected bias from public statistics and from their own information on prices. In summary, households face two different problems at the same time: (1) given that they have uncertainty about the state of inflation, they want to use all sources of information to learn about the mean inflation rate; (2) but trying to account for the fact that information sets may be biased, and not all information is equally useful. Hence, households are also interested in learning $E(b^i)$ and $E(b^p)$ for every public signal p.

Given the normality assumption about the distribution of prices, the way signals are constructed and by assuming normal orthogonal priors, it can be shown[36] that a Bayesian household form expectations in this way

$$E[\pi^{e,i}_{t+1|t}|\mathcal{I}^i_t] = \pi^{e,i}_{t|t-1}(1-\psi_i - \sum_{p=1}^{P}\psi_p) + (\pi^i_{t|t} - b^i_{t|t-1})\psi_i + \sum_{p=1}^{P}(\pi^p_{t|t} - b^p_{t|t-1})\psi_p \quad (7)$$

where $E[\pi^{e,i}_{t+1|t}|\mathcal{I}^i_t]$ represents inflation expectations for next period and are the posterior beliefs of a Bayesian learner. Inflation expectations are a convex combination of prior beliefs about inflation ($\pi^{e,i}_{t|t-1}$), public signals ($\pi^p_{t|t}$) and idiosyncratic information ($\pi^i_{t|t}$) about past inflation. However, households do not take neither the public statistics nor their idiosyncratic information "face value". They know that each piece of information may contain bias, so they subtract their percep-

[35] Cavallo et al. (2016) provides experimental evidence for Argentina in 2012, showing that on average people react sophisticatedly to bias information, subjects in the experiment de-biased the official statistics in such a way that official and unofficial signals coincide. Here I extend their main model to account for stylized facts presented in previous sections

[36] See Appendix

tion about the bias before using any piece of information, public or idiosyncratic. $b^{e,p}_{t|t-1}$ represents the prior beliefs about each public statistics p bias. $b^{e,i}_{t|t-1}$ represents prior beliefs about how representative is consumption-basket inflation with respect to the inflation rate. Perceived idiosyncratic bias is updated according to this rule

$$E[b^{e,i}_{t+1|t}|\mathcal{I}^i_t] = b^{e,i}_{t|t-1}(1-\omega_i-\sum_{p=1}^{P}\omega_p) + (\pi^i_{t|t}-\pi^{e,i}_{t|t-1})\omega_i + \sum_{p=1}^{P}(\pi^i_{t|t}-(\pi^p_{t|t}-b^{e,p}_{t|t-1}))\omega_p \tag{8}$$

Perceived public signal bias is updated according to this rule

$$\begin{aligned} E[b^{e,p}_{t+1|t}|\mathcal{I}^i_t] = b^{e,p}_{t|t-1}&(1-\phi_p-\phi_i-\sum_{\substack{j=1,\\ j\neq p}}^{P}\phi_j) \\ &+ (\pi^p_{t|t}-\pi^{e,i}_{t|t-1})\phi_p \\ &+ (\pi^p_{t|t}-(\pi^i_{t|t}-b^{e,i}_{t|t-1}))\phi_i \\ &+ \sum_{\substack{j=1,\\ j\neq p}}^{P}(\pi^p_{t|t}-(\pi^j_{t|t}-b^{e,j}_{t|t-1}))\phi_j, \qquad \forall p \end{aligned} \tag{9}$$

Information Sets.

$$\mathcal{I}^i_t = \{\pi^p_s, \pi^i_s, \sigma^2, \sigma^2_p, \sigma^2_i : s = 1, .., t; p = 1, ..., P\}$$

For the sake of simplicity, it is assumed that the inflation variance and the precision of the signals is common knowledge, i.e., households know σ^2, σ^2_g and σ^2_i. Note that even when households do not know their mean level of inflation μ_i, it is learnable by Law of Large Numbers since $\bar{\pi}^i$ is an unbiased signal of the price change of the subset of products i[37]. Note that it is possible to assume that households have some knowledge about the distribution of α_i.

IV One Biased Public Signal

I first discuss the case where there is only one public signal. Each household observes the same public signal of about the change in prices. π^g can identified

[37] By LLN $\frac{1}{t}\sum_{j=1}^{t}\bar{\pi}^i_t \to \mu_i$ as $t \to \infty$

as the inflation implied by the official consumer price index and is given by

$$\pi^g = \bar{\pi}^g + b^g \tag{10}$$

where $\bar{\pi}^g \sim \mathcal{N}(\mu,\, \sigma_g^2)$. $\bar{\pi}^g$ represents what the government observes when measuring inflation and b^g is the intentional bias the government adds to the official measure[38].

For simplicity, I assume that there exists two types of households, lower income (type L) and higher income (type H). Lower income households face a consumption-basket inflation given by

$$\pi^L \sim \mathcal{N}(\mu_L,\, \sigma_L^2)$$

while higher income households experience an average increase in prices given by

$$\pi^H \sim \mathcal{N}(\mu_H,\, \sigma_H^2)$$

I assume that $\mu_L > \mu_H$, which means that lower income households have a higher price change rate than higher income households. Because the focus of the paper is on mean effect rather than on its volatility, the benchmark specification assumes $\sigma_L^2 = \sigma_H^2$. The inflation rate in any period t is

$$\pi_t = \alpha\pi_t^L + (1-\alpha)\pi_t^H$$

in which α represents the share of lower income households' consumption on total expenditure. Note that

$$\begin{aligned} E(\pi) &= \alpha\mu_L + (1-\alpha)\mu_H \\ Var(\pi) &= \alpha^2\sigma_L^2 + (1-\alpha)^2\sigma_H^2 \end{aligned} \tag{11}$$

and

$$\begin{aligned} E(b^L) =& \mu_L - \mu = (1-\alpha)(\mu_L - \mu_H) \\ E(b^H) =& \mu_H - \mu = \alpha(\mu_H - \mu_L) \end{aligned} \tag{12}$$

Households neither observe the expected inflation in (11) nor the expected biases

[38]In principle it can also represents an unintentional bias coming from mistakes in the measurement system

in (12), they use the government signal and their own information to learn about them. In summary, household i's problem is to learn three targets out of two signals, one public and one idiosyncratic.

i Characterization: one public signal

To characterize the expectations formation process, I simulate the learning process using Monte Carlo random sampling with $1,000$ repetitions. The benchmark specification assumes $\mu = 20$, $\mu_L = 25$, $\mu_H = 15$ and $\sigma = \sigma_g^2 = \sigma_H^2 = \sigma_L^2 = 4$. The government releases a constant signal $\pi^g = 10$, so $b_t^g = 10 - \bar{\pi}_t^g$ and the true expected government bias is $E(b^g) = -10$. Initial priors are correct in the benchmark specification for both types of households, i.e., $\{\pi_0, b_0^g, b_0^i\} = \{20, (-10), |5|\}$[39]. I simulate the economy for 100 periods.

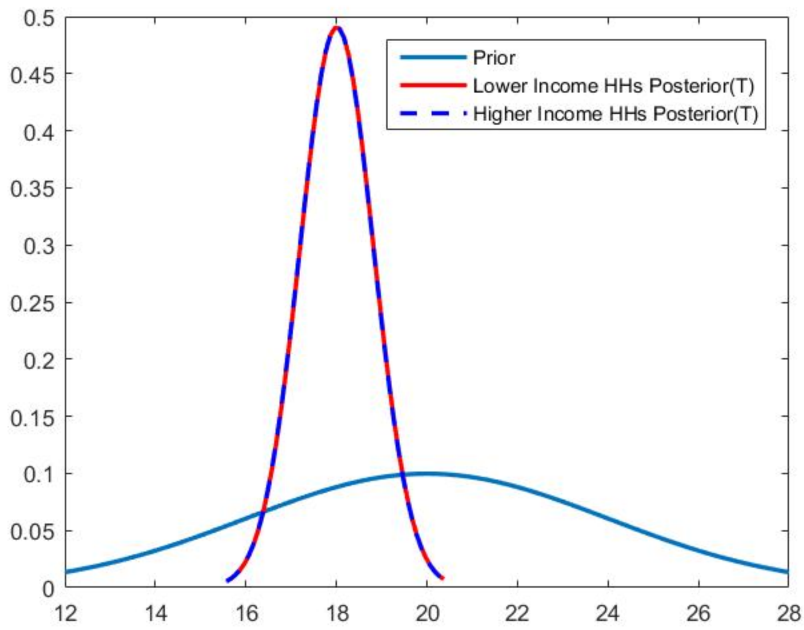
ii Does the government want to release biased information?

Assume household i's initial prior about government bias falls short and is $b_0^g = -5$ for both types (the benchmark specification in Subsection (i) holds for the remaining parameters and priors). Given the fact that households underestimate the downward bias in public information, they tend to underestimate inflation in the short and, more importantly, in the long run. Figure (4) shows the initial prior distributions and posteriors for both types of households after T periods.

Why is that? When households update their expectations with incoming signals, they realize that their perceived bias from public information is inaccurate. Hence, they increase their perceived bias as shown in Figure (5a). However, because they do not have enough information to fully learn the quality of each signal, households consider the possibility that their perceived bias about idiosyncratic information is inaccurate as well. Hence, lower income households increase their perceived idiosyncratic bias and higher income households decrease it (in absolute terms) as shown in Figure (5b). Both types of households that have a correctly prior about the quality of their own signal, act in opposite ways: lower income households perceived that their own consume experience is less valuable to predict inflation, while higher income households belief that their own signal is relatively

[39]Note that b_0^i is positive for the lower income type and negative for the higher income type.

Figure 4: Prior and Posterior Expected Inflation by Household Type



more informative. Even though they correct their public perceived bias in the right direction, it is precisely for the simultaneous update of their idiosyncratic perceived bias that they never learn the true mean of the inflation dynamics.

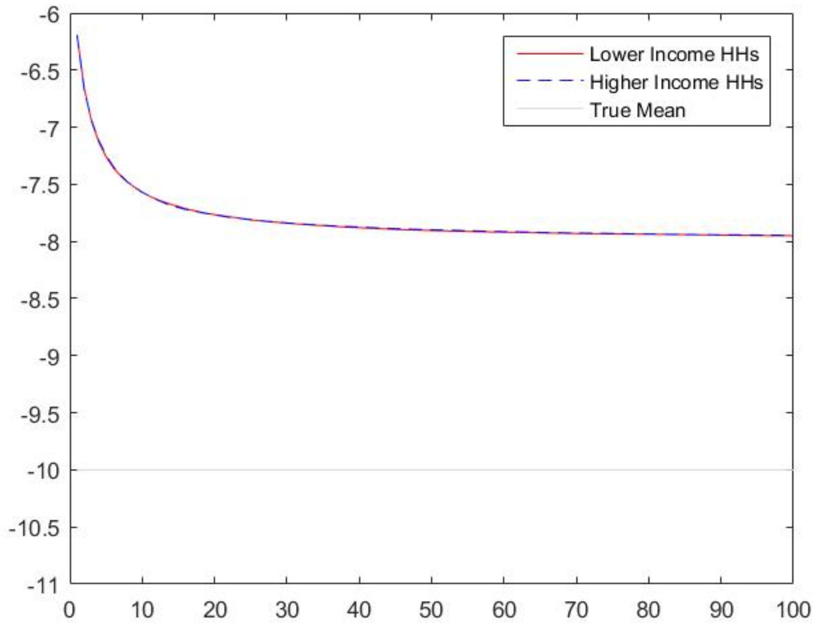
Suppose that a revenue-maximizing government increases money supply at a rate η. Because households do not fully observed the state of the economy but through signals and expectations are essentially backward-looking, the initial perception of the bias of public information is crucial. Whenever households underestimate a downward bias in public information, the government may obtain a higher revenue through a lower expected inflation.

iii Effect of Consumption Experience on Expectations

The previous subsection shows that even when there is agreement among households about the true inflation rate, learning (i.e., convergence to the true mean of the inflation dynamics) may not be possible if the initial perception about the quality of signals is not sufficiently accurate. This subsection considers under which circumstances heterogeneous expectations emerge.

Figure 5: Expectations about government and idiosyncratic bias (in absolute value)

(a)



(b)

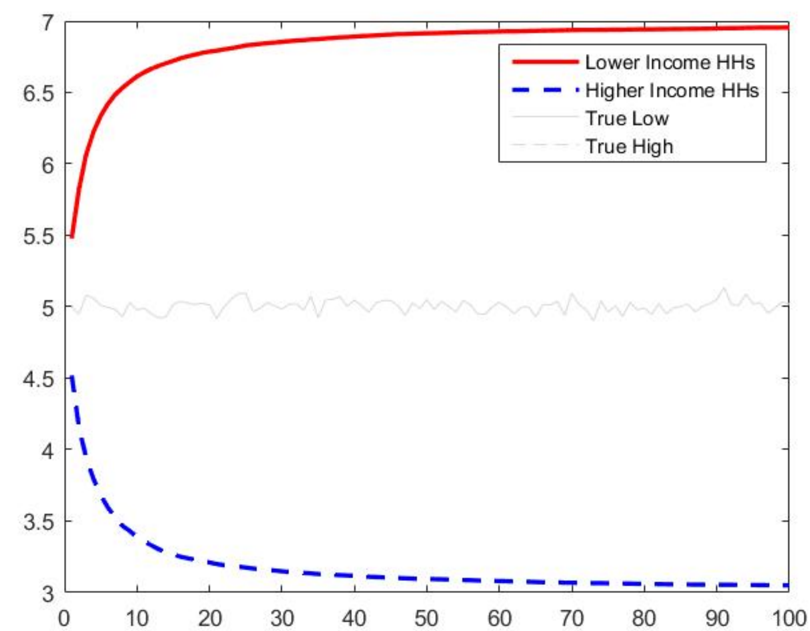
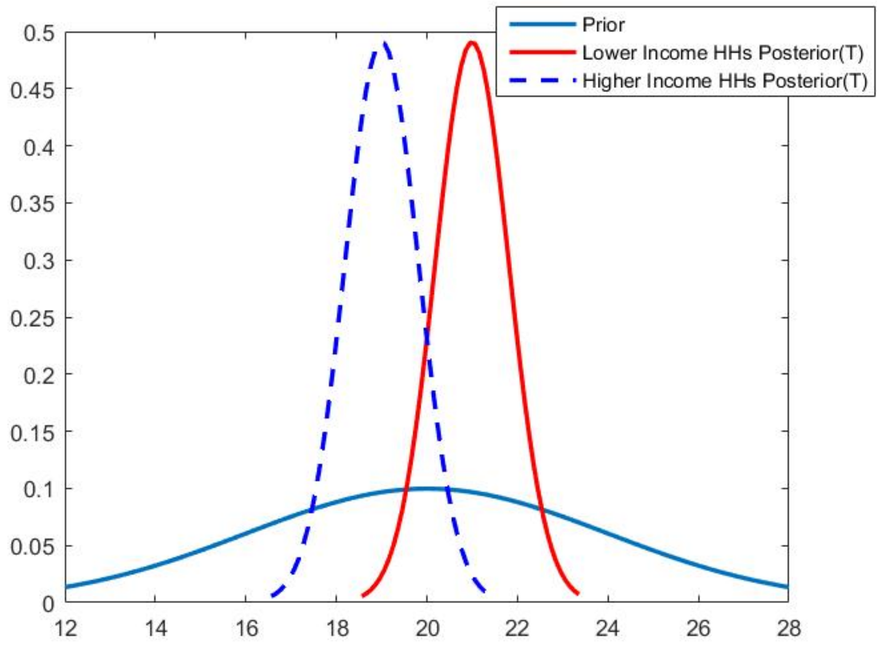


Figure 6: Prior and Posterior Expected Inflation by Household Type



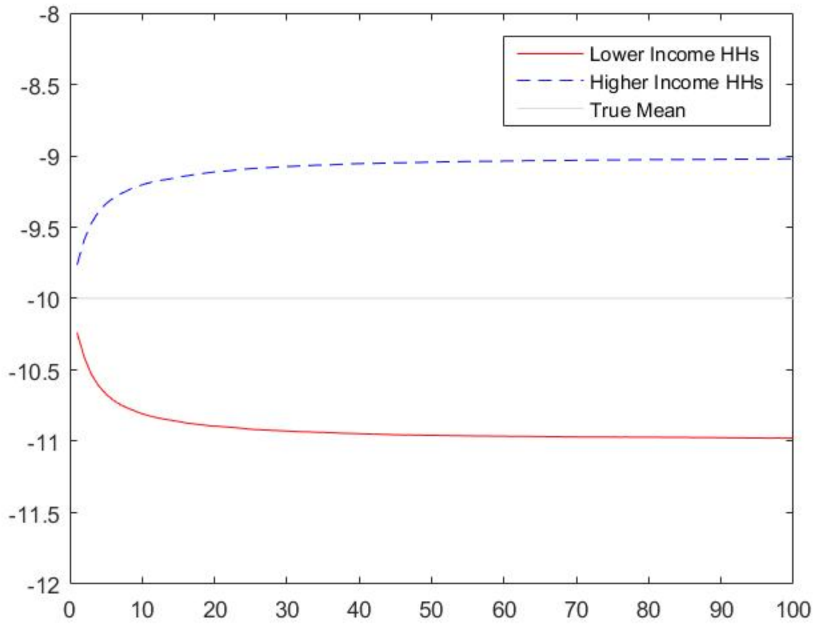
Assume household i's initial prior about their own information bias falls short in absolute terms and is $b_0^i = |2.5|$, positive for the lower income type and negative for the higher income type. Figure (6) shows the initial prior distributions and posteriors for both types of households after T periods. Although households have a correct initial perception about the quality of public information, the fact that they overestimate the quality of their own information affect their long-run learning. Both types of households update upwards the idiosyncratic perceived bias in absolute terms as shown in Figure (7b). However, they also consider the possibility that their public perceived bias is not correct. Hence, lower income households increase their public perceived bias in absolute terms, while higher income households decrease it as shown in Figure (7a).

Figure (6) shows an important property of the model. Inflations expectations may be correct on average by disagreement may be persistent in the long run due to idiosyncratic consumption experience[40].

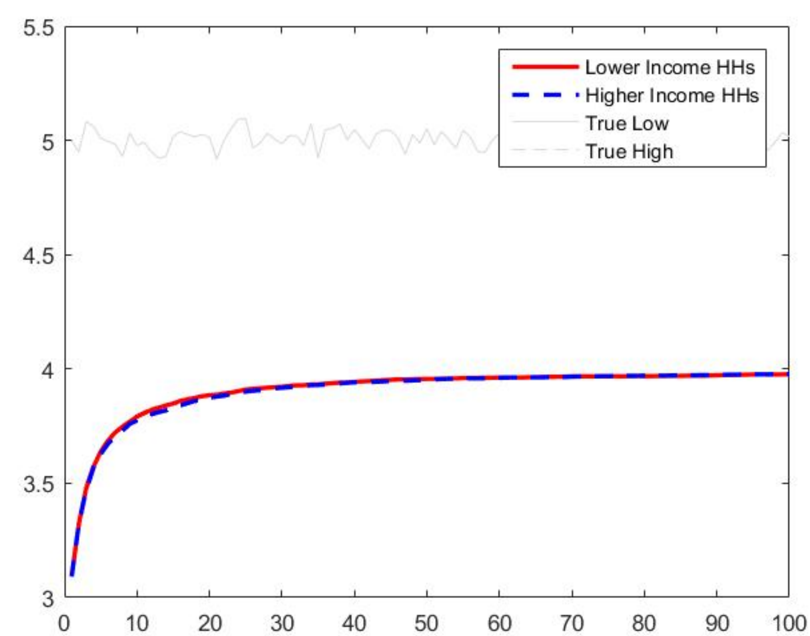
[40]Heterogeneity may also emerge if households have a different initial public perceived bias

Figure 7: Expectations about government and idiosyncratic bias (in absolute value)

(a)



(b)



V Multiple Public Signals

Assume there is an additional public signal which is unbiased. Each Bayesian household still has to learn three targets but know they receive equal number of signals. Appendix B shows how inflation expectations are form in this situation. The unbiased signals allow households to learn the true mean of the inflation dynamics and prevent heterogeneity of expectations in the long run.

%%% PLOT RESULTS HERE %%%

VI Discussion

Empirical Analysis

Substitution effect. One indirectly way to test the inter-category substitution effect within each demographic group is by exploiting the time length of the national consumer survey. Encuesta Nacional de Gastos de los Hogares (ENGH) 2004-2005 was conducted between October-2004 and December-2005. By weighting price sub-indexes of the CPI with the average consumption basket weight for each group, I can estimate the average inter-category elasticity of substitution. I can then assume the same effect for the period 2007-2011 and replicate results as a robustness check.

Inflation expectations. The publicly available information includes the mean and median of households' answers, grouped by the level of education (two levels) and by home location (three regions). The micro-data of the survey would allow me to estimate a reduced-form model with more specific pseudo-groups to enrich the empirical results.

VII Conclusion

The paper shows that, under uncertainty about the current aggregate level of prices, individuals use their own consumption experience (price information from their consumption basket) both to decide their set of relevant contemporaneous information and to form inflation expectations.

Even when the expected average inflation closely followed only one of the sources of public information (unofficial statistics), heterogeneous inflation expectations emerged as a result of the use of idiosyncratic information. The group of households with a lower level of education expected a higher inflation rate than the group of households with a higher level of education in the period considered. I find that this pattern is correlated with the consumption-basket inflation rate experienced by each group. In particular, food prices increased faster than the average increase in prices during the period under study and are overrepresented in the lower educated consumption baskets.

Differences in relative price movements are likely to explain an important part of the heterogeneity in inflation expectations. A result that differ from previous studies (e.g. Madeira & Zafar, 2015; Hobijn et al., 2009; McGranahan & Paulson, 2006), but it is consistent with having uncertainty about the quality of public inflation statistics in a moderately high inflation environment. Empirical results suggest that heterogeneity on inflation expectations may highly depend on some representative prices.

The Bayesian learning model here proposed allows to disentangle the effect of information uncertainty from the increase in relative price dispersion. It also helps to explain under which conditions learning and agreement about the expected inflation rate is possible among different types of households. The model shows that whenever households use their own information on prices, the existence of unbiased sources of information is not sufficient; a correct perception about its quality is also necessary.

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Appendix

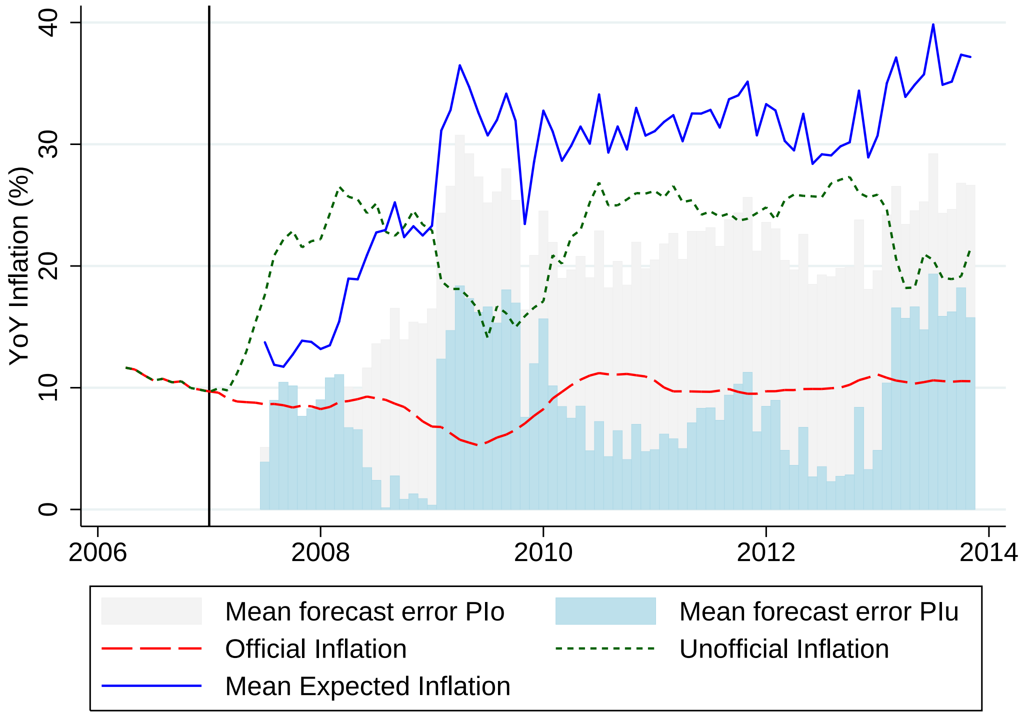
Appendix A: Empirical Analysis, data definitions and sources

i Data sources and definitions

National Consumer Survey: Encuesta Nacional de Gasto de los Hogares 2004-2005

ii Evidence on multiple information sets

Figure 8: Official and Unofficial Forecast Errors Against One Year Ahead Expected Inflation, 2007-2013



Note: Expected Inflation is from the Encuesta de Expectativas de Inflación (UTDT). The official CPI is IPC INDEC (2008=100) and the unofficial CPI is the CPI computed by Price Stats (Inflación Verdadera)

iii Consumption-basket inflation

The approach to identify household's consumption-basket inflation rates is summarized as follows. I combine the Cavallo (2013)'s approach to compute price indexes of different categories by using online prices with household's expenditure weights from the national consumer survey. The first stage is to organize online prices from the scrape data collected by Billion Price Project to compute categories price indexes. The unweighted geometric mean of relative prices of each category j per day t is computed according to

$$UGM_t^j = \prod_g (p_{t+1}^g / p_t^g)^{1/n_t^j} \tag{13}$$

where p_t^g is the price of good g at time t and n_t^j is the number of goods from which information is collected in a particular day.

The second stage is to compute a category index j at t:

$$I_t^j = UGM_0^j.UGM_1^j...UGM_{t-1}^j \tag{14}$$

This procedure is identical to the one performed by Cavallo (2013). See his paper for details.

Using the classification of the Bureau of Labor Statistics, the third stage is to match price categories from the Billion Price Project data base with the categories reported in the Encuesta Nacional del Gasto de los Hogares 2004-2005's consumer survey. I match a total of 53 categories.

The next step is to compute household h weights by using the expenditure information from the national consumer survey Encuesta Nacional de los Hogares 2004-2005. Weights are constructed according to

$$w_j^h = \frac{E_j^h}{\sum_{j=1}^C E_j^h} \quad with \quad \sum_{j=1}^C w_j^h = 1 \tag{15}$$

where w_j^h is the weight of category j in household h consumption basket, and E_j^h is the total expenditure of household h in category j by the time the consumer survey was conducted. Note that E_j^h may include the expenditure done by the household as a unit plus any expenditure may by individual members on their own. For each household I compute the weights $j = 1, ..., C$, where C represents the total number of categories that coincide with those provided by the Billion Price Project data base. The consumer survey includes around $26,000$ different households.

Finally, I compute a household price index for each h as the weighted arithmetic mean of all category I_t^j indexes, with weights given by w_j^h for $j = 1, ..., C$. By calculating the growth rate of the household h index, I obtain the household's consumption-basket inflation rates.

Table (3) shows the average weights of each category on households' consumption baskets by the level of education; categories are ranked by the inflation experienced between October 7th 2007 and March 24th 2011. The 10 categories that experienced the highest level of inflation during the period represented 27.6% and 22.7% of the consumption baskets of the households with a lower and a higher level of education, respectively. When we consider the sample of the first 25 categories, they represent 52.5% of the average consumption basket of households with a lower level of education and 46.5% of the average consumption basket of households with a higher level of education.

The average across households of the median annual daily inflation for interquartile range is 20.31% with a standard deviation of 0.86. The average across households of the total cumulative inflation for interquartile range is 95.85% with a standard deviation of 7.11.

Table 3: Households' Consumption Basket Weights and Relative Discrepancy by Level of Education

Category Name	Index	Mean LE	Mean HE	Price Change	Weights	Impact of Category
UNCOOKED BEEF STEAKS	319.6	11.7	7.4	0.6713	0.5750	2.8667
LAMB, ORGAN MEATS, AND GAME	254.3	0.2	0.1	0.3299	0.7311	0.0274
UNCOOKED GROUND BEEF	249.4	0.4	0.7	0.3042	-0.3999	-0.0890
SAUSAGES	232.2	1.2	1.1	0.2142	0.0812	0.0195
HAM	229.1	1	1.3	0.1981	-0.2588	-0.0659
FRESH FISH AND SEAFOOD	219.2	0.4	0.5	0.1460	-0.2583	-0.0193
RICE	212.5	1.4	0.8	0.1113	0.7034	0.0639
BREAD	207.1	8	5	0.0828	0.5922	0.2459
TIRES	206.8	0.3	0.6	0.0813	-0.5533	-0.0279
CHEESE AND RELATED PRODUCTS	206.0	3	4.3	0.0770	-0.3082	-0.1026
PASTA	204.5	3.5	3	0.0693	0.1504	0.0315
CHICKEN	203.3	4.3	2.8	0.0631	0.5368	0.0946
PROCESSED FISH AND SEAFOOD	203.0	0.2	0.4	0.0613	-0.5754	-0.0144
EGGS	201.2	2.1	1.5	0.0518	0.3934	0.0307
CARBONATED DRINKS	198.5	5.2	5.6	0.0378	-0.0745	-0.0158
SUGAR AND ARTIFICIAL SWEETENERS	198.4	1.5	0.8	0.0374	0.9157	0.0269
CANDY AND CHEWING GUM	197.5	0.5	0.6	0.0325	-0.2469	-0.0049
HOUSEHOLD PAPER PRODUCTS	197.3	0.6	1	0.0316	-0.4123	-0.0133
SWEETROLLS, COFFEE CAKE & DOUGHNUTS	196.6	0.7	1.1	0.0278	-0.3710	-0.0111
OTHER CONDIMENTS	196.5	0.4	0.5	0.0273	-0.1002	-0.0014
MILK	195.7	3.5	3.7	0.0235	-0.0535	-0.0046
BEER AT HOME	193.7	0.8	1.1	0.0129	-0.2142	-0.0030
CHOCOLATE	191.6	0.1	0.3	0.0020	-0.4503	-0.0002
SALT AND OTHER SEASONINGS AND SPICES	190.8	0.1	0.1	-0.0023	0.6849	-0.0001
CANNED FRUITS AND VEGETABLES	189.8	1.2	1.1	-0.0076	0.1807	-0.0015
TEA	189.4	1.7	1.3	-0.0098	0.3162	-0.0039
FRESH BISCUITS, ROLLS	188.6	1.5	2.1	-0.0139	-0.2980	0.0086
PAINT, WALLPAPER TOOLS & SUPPLIES	185.6	0.9	1.2	-0.0294	-0.2581	0.0093
SOUPS	185.5	0.3	0.4	-0.0301	-0.1363	0.0014
FLOUR AND PREPARED FLOUR MIXES	184.7	1.1	0.4	-0.0341	1.8221	-0.0252
OTHER DAIRY AND RELATED PRODUCTS	180.5	3.3	5.3	-0.0564	-0.3831	0.1149
DISTILLED SPIRITS AT HOME	180.0	0.3	0.3	-0.0589	-0.2544	0.0051
WINE AT HOME	177.9	1.9	2.3	-0.0697	-0.1754	0.0277
NONFROZEN NONCARBONATED JUICES AND DRINKS	177.0	1.4	1.4	-0.0743	-0.0098	0.0010
SHAMPOO, BATH PRODUCTS	176.0	1.4	1.9	-0.0797	-0.2362	0.0357
MISCELLANEOUS HOUSEHOLD PRODUCTS	171.9	0.5	0.7	-0.1011	-0.2623	0.0177
CLEANING PRODUCTS	170.5	5.6	5.8	-0.1083	-0.0391	0.0246
DEODORANT/SUNTAN PREPARATIONS	167.6	0.7	0.9	-0.1237	-0.3012	0.0348
PERFUME	167.2	0.3	0.4	-0.1256	-0.3647	0.0189
DENTAL & NONELECTRIC SHAVING PRODUCTS	166.3	0.8	1.1	-0.1307	-0.2349	0.0338
SANITARY/FOOTCARE PRODUCTS	165.6	1.8	2.2	-0.1340	-0.1615	0.0475
BABY CARE PRODUCTS	164.9	1.3	1.2	-0.1377	0.0672	-0.0115
VEHICLE PARTS & EQUIPMENT OTHER THAN TIRES	164.5	0.3	0.7	-0.1397	-0.5441	0.0507
OTHER FRESH VEGETABLES INCLUDING FRESH HERBS	164.2	7.8	5.9	-0.1413	0.3103	-0.2596
OTHER FRESH FRUITS	161.3	3.1	3.8	-0.1568	-0.2005	0.1211
COSMETICS NAIL PREPARATIONS & IMPLEMENTS	159.0	0.4	0.9	-0.1689	-0.4954	0.0726
OVER-THE-COUNTER DRUGS	158.4	5.3	5.8	-0.1716	-0.0848	0.0841
OTHER FATS AND OILS	150.9	1.7	1	-0.2111	0.7398	-0.1489
TOYS, GAMES, HOBBIES, & PLAYGROUND EQUIPMENT	144.8	0.1	0.3	-0.2427	-0.4860	0.0299
TOOLS	141.8	0.1	0.2	-0.2585	-0.3579	0.0166
BOOKS	131.8	0.9	1.8	-0.3110	-0.4918	0.2790
HOME FURNITURE	128.6	1.5	2.1	-0.3274	-0.3139	0.2199
APPLIANCES	128.2	1.8	3.2	-0.3296	-0.4344	0.4622
TOTAL	191	100	100	0	-	4.3

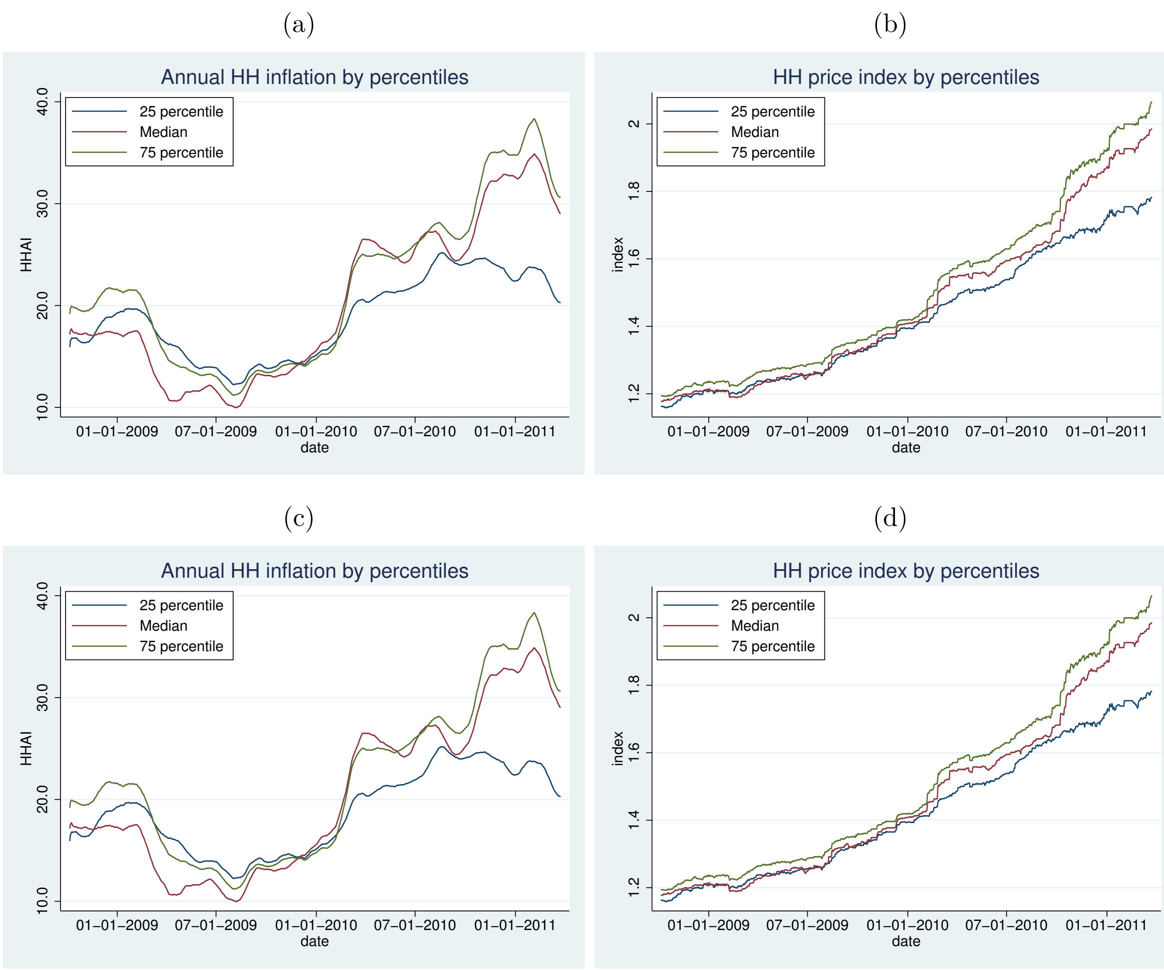
Table (3) also reports a decomposition (Hagemann, 1982) for the relative discrepancy between a CPI constructed with the average weights expenditures of lower educated households and a CPI constructed with the average weights expenditures of higher educated households. The relative discrepancy is measured by

$$Relative\ Discrepancy\ by\ Education = \left(\frac{I_t^j}{I_t^{CB_H}} - 1\right)\left(\frac{w_j^L}{w_j^H} - 1\right)\left(w_j^H * 100\right)$$

The relative discrepancy between group-specific CPIs reaches 4.3%.

Stability of the relationship over time

Figure 9: Dynamics of the annual inflation rates and price indexes by selected percentiles households



I measure how stable is the relationship over time by plotting the evolution of the consumption-basket inflation rate for selected households. Additionally, I construct a transition matrix by quintiles to check the probability of a household with a certain consumption-basket inflation rate in December 2008 to be in the same position after two years.

Table 4: Household Relative Position on the Consumption Baskets Inflation Distribution. Transition Matrix by Quintiles.

	Index Dec-2010				
Index Dec-2008	1st	2nd	3rd	4th	5th
1st	66.6	14.7	8.9	6.4	3.4
2nd	21.7	24.8	18.6	18.5	16.5
3rd	8.1	22.0	20.6	23.5	25.8
4th	3.3	22.4	20.9	24.6	28.8
5th	0.4	16.1	30.9	26.9	25.6

Notes: Households are increasingly ordered by their respectively constructed inflation price index at the end of the year 2008. The transition matrix measures the position of the same households at the end of the year 2010.

More on the idiosyncratic information on prices by level of education

The number of households with the lower level of education represents the 80.4% of the sample. Their average mean annual daily consumption-basket inflation is 20.45%, with a standard deviation of 2.48 and a 95% confidence interval of 20.41% − 20.49%. The average mean annual daily consumption-basket inflation for households with higher level of education is 19.33%, with a standard deviation of 2.61 and a 95% confidence interval of 19.26% − 19.40%. The evidence is not affected by outliers households, it robust to the interquatile range with an average mean annual daily consumption-basket inflation of 20.35% and 20.16% and standard deviation of 0.85, respectively. In both cases, a two-sample t-test with unequal variances shows that the difference in means is statistically significantly different than 0 at 1%.

Another way to see how different are the consumption-basket inflation rates experienced by both groups of households is by regressing actual inflation rates on the level of education. Table (5) shows the cross-sectional regressions of the household mean annual inflation and household total cumulative inflation, for the Period 2007-2011, on the level of education. Table (6) presents a similar analysis but exploits the constructed panel on household price indexes. Households with lower level of education experienced higher consumption-basket inflation rates. Results are statistically significant at 1%.

Table 5: Cross-sectional Regressions of the Household Mean Annual Inflation and Household Cumulative Inflation, for the Period 2007-2011, on the Level of Education.

	(1) $\overline{\Pi}$	(2) $\overline{\Pi}$	(3) $\overline{\Pi}$	(4) $\Pi^{2007-2011}$	(5) $\Pi^{2007-2011}$	(6) $\Pi^{2007-2011}$
Low Educated	1.122***			8.224***		
	(0.0393)			(0.310)		
Kinder		0.741	0.733		5.474	5.431
		(1.249)	(1.244)		(9.865)	(9.817)
Primary		1.316***	1.162***		9.528***	8.196***
		(0.0416)	(0.0427)		(0.328)	(0.337)
Secondary		0.822***	0.717***		6.203***	5.285***
		(0.0447)	(0.0452)		(0.353)	(0.356)
Greater BA			0.653***			5.891***
			(0.0656)			(0.518)
Rest of ARG			0.781***			6.754***
			(0.0539)			(0.426)
Constant	19.33***	19.33***	18.75***	91.25***	91.25***	86.23***
	(0.0352)	(0.0351)	(0.0535)	(0.278)	(0.277)	(0.423)
Observations	25833	25833	25833	25833	25833	25833

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Panel Regressions of Household Price Indexes on the Level of Education.

	(1)	(2)	(3)
	p_t^i	p_t^i	p_t^i
Low Educated	0.0211***		
	(0.000862)		
Kinder		0.0212***	0.0211**
		(0.00669)	(0.0101)
Primary		0.0245***	0.0213***
		(0.000901)	(0.000920)
Secondary		0.0157***	0.0135***
		(0.000980)	(0.000984)
Greater BA			0.0139***
			(0.00136)
Rest of ARG			0.0162***
			(0.00116)
Constant	1.349***	1.349***	1.337***
	(0.000785)	(0.000785)	(0.00118)
Observations	32678745	32678745	32678745

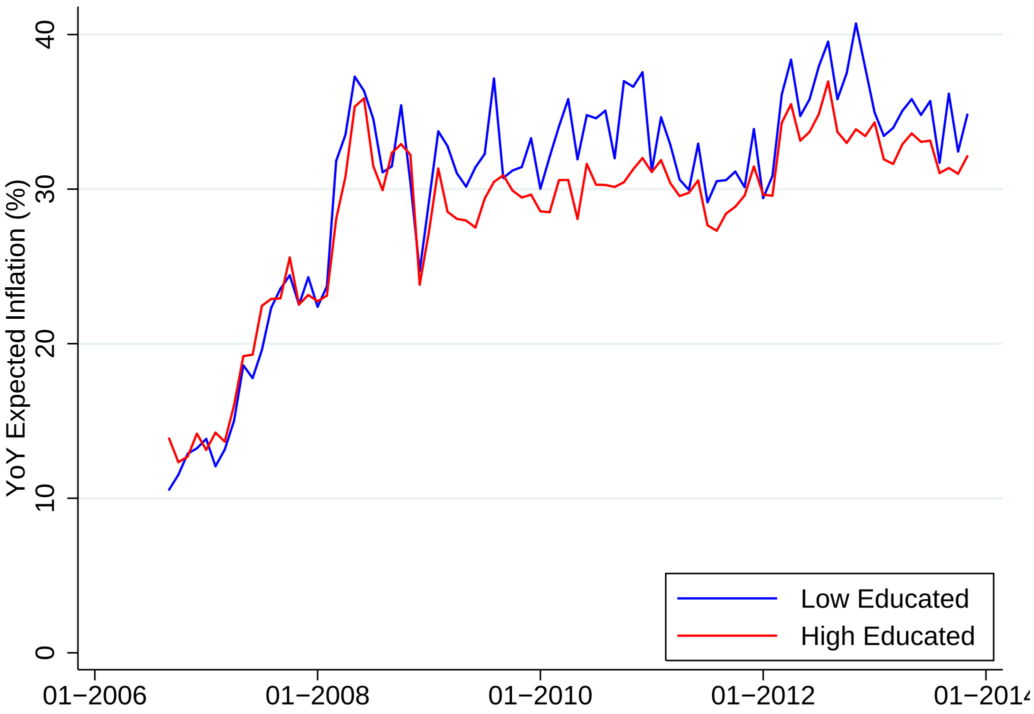
Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

iv Households' inflation expectations

A comparison of the time series of average expected inflation and the average realized inflation rate of the consumption basket shows a simple correlation of 0.38 and 0.31 for $\pi^{e,H}_{t+12|t}$ and $\pi^{CB_H}_{t-1}$ and $\pi^{e,H}_{t+12|t}$ and $\pi^{CB_H}_{t-2}$, respectively, for households with higher education. Simple correlations for lower educated households are 0.31 and 0.16, respectively.

Figure 10: Mean Annual Expected Inflation by Education, 2007-2013



Note: Expected Inflation is from the Encuesta de Expectativas de Inflación (UTDT).

v Equivalence Between Consumption Baskets Categories

Table 7: Equivalence Between Consumption Baskets Categories

BBP Description	Official CPI Description	Consumer Survey Description
FLOUR AND PREPARED FLOUR MIXES	Harina de trigo	Harina de trigo
	Otras harinas	Harina de maíz
PASTA	Fideos secos	Fideos secos
	Pastas frecas	Fideos frescos
		Ñoquis frescos
		Ravioles frescos
	Tapas de masa	Tapas frescas para empanadas y pastelitos
		Tapas frescas para tartas
	Semipreparados en base a pastas	Pre-pizza
RICE	Arroz	Arroz blanco
BREAD	Pan Fresco	Pan tipo francés fresco en piezas
	Pan Envasado	Pan envasado en rebanadas blanco
		Pan envasado en rebanadas integral
		Pan para hamburguesas - pebetes - panchos
		Pan rallado
FRESH BISCUITS, ROLLS	Galletitas dulces y otros	Galletitas dulces envasadas

table continued ...

BBP Description	Official CPI Description	Consumer Survey Description
		Alfajores
SWEETROLLS, COFFEE CAKE & DOUGHNUTS	Facturas	Facturas y churros
UNCOOKED GROUND BEEF	Semipreparados en base a carne vacuna	Hamburguesas para cocinar (semipreparados)
UNCOOKED BEEF STEAKS	Cortes delanteros y traseros de carne vacuna fresca	Asado Bife ancho Carnaza común Cuadrada Falda Hueso con carne Matambre - cima Nalga Paleta Roast beef Vacío Otros cortes
HAM	Fiambres y conservas	Jamón cocido Mortadela Paleta (fiambres) Salame
SAUSAGES	Embutidos frescos	Chorizo fresco Morcilla Salchicha tipo viena con y sin piel
LAMB, ORGAN MEATS, AND GAME	Menudencias y achuras frescas vacunas	Lengua de vaca Mondongo Riñón Otras achuras y menudencias vacunas
CHICKEN	Aves frescas y congeladas	Pollo entero
FRESH FISH AND SEAFOOD	Pescados frecos y congelados	Merluza
PROCESSED FISH AND SEAFOOD	Conservas de pescado	Atún en conserva
EGGS	HUEVOS	Huevos de gallina
MILK	Leche fluida	Leche común entera Leche común descremada Otras leches fluidas
	Leche en polvo	Leche en polvo entera o descremada
CHEESE AND RELATED PRODUCTS	Quesos blandos y untables	Queso crema Queso doble crema - cuartirolo Queso port salut
	Quesos semicrudos	Queso paté-gras - mar del plata
	Quesos duros	Queso para rallar Queso rallado
OTHER DAIRY AND RELATED PRODUCTS	Manteca y crema	Manteca Crema de leche
	Yogur y postres lacteos	Yogur natural o saborizado Yogur con aditamentos Dulce de leche
	Helados	Helado individual Helado envasado por kilo o litro Helado suelto
OTHER FRESH FRUITS	FRUTAS FRESCAS	Ananá Banana Ciruela Durazno Frutillas frescas Kiwi fresco Limón

table continued ...

BBP Description	Official CPI Description	Consumer Survey Description
		Mandarina Manzana Melón fresco Naranja Pera Pomelo fresco Uva fresca
OTHER FRESH VEGETABLES INCLUDING FRESH HERBS	VERDURAS FRESCAS	Acelga fresca Ají fresco Ajo Apio, hinojo fresco Batata fresca Berenjenas frescas Cebolla común fresca Cebolla de verdeo, puerro fresco Coliflor, brócoli fresco Chauchas frescas Choclo fresco Lechuga fresca Papa fresca Pepino fresco Remolacha fresca Repollo fresco Tomate perita fresco Tomate redondo fresco Zanahoria fresca Zapallitos frescos Zapallo fresco
CANNED FRUITS AND VEGETABLES	FRUTAS EN CONSERVA Tomate en conserva Otras verduras secas y en conserva	Aceitunas Duraznos en almíbar o en conserva Tomates en conserva Lentejas secas Arvejas en conserva
CARBONATED DRINKS	BEBIDAS GASEOSAS	Gaseosas
NONFROZEN NONCARBONATED JUICES AND DRINKS	JUGOS Y REFRESCOS	Aperitivo sin alcohol Jugos y refrescos en polvo para preparar bebidas Jugos y refrescos líquidos para preparar bebidas Jugos y refrescos para beber sin diluir
TEA	YERBA MATE TÉ	Yerba mate Mate cocido en saquitos Té común en saquitos
SUGAR AND ARTIFICIAL SWEETENERS	Azúcar y edulcorantes	Azúcar Edulcorante
CANDY AND CHEWING GUM	Caramelos y golosinas	Caramelos o confites Chicles Pastillas en paquete
CHOCOLATE	Cacao y derivados	Chocolate para taza/repostería Cacao azucarado o no
OTHER FATS AND OILS	Aceites puros Aceites mezcla	Aceite de girasol Aceite mezcla
SOUPS	SOPAS Y CALDOS CONCENTRADOS	Caldos concentrados Sopas concentradas
SALT AND OTHER SEASONINGS AND SPICES	SAL Y OTROS CONDIMENTOS	Sal fina
OTHER CONDIMENTS	ADEREZOS	Mayonesa Vinagre, aceto balsámico

table continued ...

BBP Description	Official CPI Description	Consumer Survey Description
BEER AT HOME	CERVEZA	Cerveza
DISTILLED SPIRITS AT HOME	OTRAS BEBIDAS ALCOHÓLICAS	Aperitivos Whisky Sidra
WINE AT HOME	VINO	Vino común Vino fino
DENTAL & NONELECTRIC SHAVING PRODUCTS	Dentífrico Máquina de afeitar descartable	Dentífrico Hoja de afeitar, repuesto para máquina de afeitar, etc.
DEODORANT/SUNTAN PREPARATIONS	Desodorante	Desodorante
COSMETICS NAIL PREPARATIONS & IMPLEMENTS	Artículos de belleza	Crema de belleza, de manos, de limpieza, nutritiva, etc. Tintura, matizador, oxidante
SHAMPOO, BATH PRODUCTS	Crema de enjuague Champú	Champú, crema de enjuague, acondicionador Champú, crema de enjuague, acondicionador
BABY CARE PRODUCTS	Pañales descartables	Pañales descartables para bebés
SANITARY/FOOTCARE PRODUCTS	Jabón de tocador Papel higiénico Tampones	Jabón de tocador Papel higiénico Toallas higiénicas, tampones, protectores diarios
PERFUME	Colonia	Colonia, loción, perfume
PAINT, WALLPAPER TOOLS & SUPPLIES	MATERIALES PARA REPARACIONES	Arena/canto rodado Cal, cemento Ladrillo Cañerías y sus accesorios Artefactos sanitarios Carpintería metálica y de madera, ventanas, puertas, etc. Membrana para techo, tejas, techado asfáltico Pisos, zócalos, revestimientos Pintura y accesorios
TOOLS	HERRAMIENTAS Y ARTÍCULOS DE FERRETERÍA	Taladro, sierra, pulidora Artículos de ferretería
CLEANING PRODUCTS	Jabones y detergentes Desengrasantes y desinfectantes Otros productos para mantenimiento y limpieza UTENSILLOS DE LIMPIEZA ARTÍCULOS DESCARTABLES	Acondicionadores y suavizantes para ropa Detergente liquido para vajilla Jabón en pan Jabón en polvo para ropa Desengrasante, desinfectante, líquidos limpiadores Lavandina Cera, brillo para pisos Insecticidas varios Balde, palangana, cestos Escoba, escobillón, plumero, lampazo, secador, etc. Esponja, lana de acero, estropajo Trapo de piso, rejilla, patines, gamuzas Servilletas de papel y rollos de papel Bolsas para residuos

table continued ...

BBP Description	Official CPI Description	Consumer Survey Description
HOUSEHOLD PAPER PRODUCTS	ARTÍCULOS DE LIBRERÍA	Anotador, block, repuesto para carpeta, resma
		Bolígrafo, lapicera, roller, lápiz mecánico, porta mina
		Carpetas
		Cuaderno
		Papel para forrar, glacé, cartulina, etiquetas, sobres, etc.
MISCELLANEOUS HOUSEHOLD PRODUCTS	BATERÍÁ DE COCINA, CUBIERTOS Y UTENSILLOS	Cacerola, olla, jarro, hervidor, colador
		Pava, cafetera, tetera (no eléctricos)
		Mamadera, utensilios para bebé
	VAJILLA Y OTROS	Copas y vasos sueltos o en juego
		Platos sueltos o en juego
TIRES	Cubiertas	Cubiertas, neumáticos
VEHICLE PARTS	Filtro de aceite	Repuestos y accesorios para el vehículo de uso del hogar
& EQUIPMENT OTHER THAN TIRES	Filtro de aire	Repuestos y accesorios para el vehículo de uso del hogar
	Pastillas de freno	Repuestos y accesorios para el vehículo de uso del hogar
	Correa de alternador	Repuestos y accesorios para el vehículo de uso del hogar
	Disco de embrague	Repuestos y accesorios para el vehículo de uso del hogar
	Bomba de agua	Repuestos y accesorios para el vehículo de uso del hogar
	Batería para automóvil	Baterías
TOYS, GAMES, HOBBIES	JUGUETES Y JUEGOS	Juguetes a cuerda, a fricción, eléctricos o a pilas
OVER-THE-COUNTER DRUGS	Analgésicos	Fiebre o dolor
	Antibióticos	Antibióticos y antisépticos
	Antiinflamatorios	Antiinflamatorios
	Cardiovasculares	Cardíacos
	Dígestivos	Digestivos
	Psicofármacos	Otros medicamentos
	Antihistamínicos	Tos
	Otros medicamentos	Dermatológicos
		Tiroides y hormonas
	ELEMENTOS PARA PRIMEROS AUXILIOS	Alcohol, agua oxigenada, antisépticos, desinfectantes
		Algodón
		Gasas y vendas
		Preservativos
APPLIANCES	ARTEFACTOS PARA COCINAR O CONSERVAR ALIMENTOS	Cocina a gas, anafe
		Cocina a otros combustibles
		Microondas
		Heladera con freezer
	ARTEFACTOS PARA LAVADO Y LIMPIEZA	Calefón, termotanque a gas
		Calefón, termotanque a otros combustibles
		Lavarropa y lavasecarropa
	ARTEFACTOS PARA EL CONFORT AMBIENTAL	Acondicionador de aire
		Estufa, calefactor eléctrico
		Ventilador de mesa o de pie, turbo circulador
BOOKS	Libros de estudio	Diccionarios, enciclopedias, atlas
		Textos primarios y de EGB
		Textos secundarios y de polimodal
		Fotocopias

table continued ...

BBP Description	Official CPI Description	Consumer Survey Description
HOME FURNITURE	Muebles	Cama
		Placard, ropero
		Cuna, moisés
		Juego de comedor
		Mesas
		Sillas, banquetas, bancos
		Modular, bar, aparador, vitrina, biblioteca
	Colchones y almohadas	Colchón o somieres

Appendix B: Theoretical Model

vi Bayesian Learning Model

This section extends the model proposed by Cavallo et al. (2016). To this end, and in order to characterize inflation expectations with a closed-form expression, I apply a conjugate Bayesian analysis of a multivariate Normal distribution. Theorem 1 states a general result.

Theorem 1. *Given prior beliefs of the vector $\vec{x}$ and a sample of signals $\vec{y}$, with marginal distribution $p(\vec{x})$ and conditional distribution $p(\vec{y}|\vec{x})$ of the form*

$$p(\vec{x}) \sim \mathcal{N}(\boldsymbol{\mu},\, \boldsymbol{\Lambda}^{-1}) \tag{16a}$$

$$p(\vec{y}|\vec{x}) \sim \mathcal{N}(\boldsymbol{A}\vec{x} + \vec{b},\, \boldsymbol{L}^{-1}) \tag{16b}$$

the conditional distribution of $\vec{x}$ after observing the sample of signals $\vec{y}$ (posterior distribution) and the marginal distribution of $\vec{y}$ are given by

$$p(\vec{x}|\vec{y}) \sim \mathcal{N}(\boldsymbol{\Sigma}\{\boldsymbol{A}^T\boldsymbol{L}(\vec{y} - \vec{b}) + \boldsymbol{\Lambda}\boldsymbol{\mu}\},\, \boldsymbol{\Sigma}) \tag{17a}$$

$$p(\vec{y}) \sim \mathcal{N}(\boldsymbol{A}\boldsymbol{\mu} + \vec{b},\, \boldsymbol{L}^{-1} + \boldsymbol{A}\Lambda^{-1}\boldsymbol{A}^T) \tag{17b}$$

where

$$\boldsymbol{\Sigma} = (\boldsymbol{\Lambda} + \boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1} \tag{18}$$

and if $\vec{x}$ has dimensionality M and $\vec{y}$ has dimensionality D, then the matrix $\boldsymbol{A}$ has size $D \times M$.

Proof. See Bishop (2006). □

Recursive application.

Proposition 2. *Let $\boldsymbol{Y} = (\vec{y}_1, ..., \vec{y}_t)$ a history of signals up to period t, where $\boldsymbol{Y}$ has dimensionality $D \times t$. Given marginal distribution $p(\vec{x})$, conditional distribution $p(\vec{y}|\vec{x})$ and $\vec{b} = 0$ of Theorem 1, the posterior update after successive t periods is*

$$p(\vec{x}|\boldsymbol{Y}, \boldsymbol{L}^{-1}) \sim \mathcal{N}(\boldsymbol{\mu}_t,\, \boldsymbol{\Lambda}_t^{-1}) \tag{19}$$

and the predictive posterior is

$$p(\vec{y^*}|\boldsymbol{Y}) \sim \mathcal{N}(\boldsymbol{A}\boldsymbol{\mu}_t,\ \boldsymbol{A}\boldsymbol{\Lambda}_t^{-1}\boldsymbol{A}^T + \boldsymbol{L}^{-1}) \tag{20}$$

where

$$\boldsymbol{\mu}_t = \qquad (\boldsymbol{\Lambda}_0 + t\boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1}(t\boldsymbol{A}^T\boldsymbol{L}\bar{\vec{y}} + \boldsymbol{\Lambda}_0\boldsymbol{\mu}_0) \tag{21}$$

$$\boldsymbol{\Lambda}_t^{-1} = \qquad (\boldsymbol{\Lambda}_0 + t\boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1} \tag{22}$$

Proof. Given initial priors $\boldsymbol{\mu}_0$ and $\boldsymbol{\Lambda}_0^{-1}$, Theorem 1 implies

$$\boldsymbol{\Sigma}_0 = (\boldsymbol{\Lambda}_0 + \boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1}$$

Note that $\boldsymbol{\Sigma}_0$ is the posterior variance of period 0 and the prior variance of period 1. Hence, $\boldsymbol{\Lambda}_1^{-1} = \boldsymbol{\Sigma}_0$, which also implies $\boldsymbol{\Lambda}_1 = \boldsymbol{\Sigma}_0^{-1}$. Similarly,

$$\begin{aligned}
\boldsymbol{\Sigma}_1 &= (\boldsymbol{\Lambda}_1 + \boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1} \\
&= (\boldsymbol{\Sigma}_0^{-1} + \boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1} \\
&= [(\boldsymbol{\Lambda}_0 + \boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A}) + \boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A}]^{-1} \\
\boldsymbol{\Lambda}_2^{-1} &= (\boldsymbol{\Lambda}_0 + 2\boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1}
\end{aligned}$$

where the last equality follows from the fact that the posterior of period t is the prior of period $t+1$. Similarly,

$$\begin{aligned}
\boldsymbol{\Lambda}_3^{-1} =&(\boldsymbol{\Lambda}_0 + 3\boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1} \\
&\vdots \\
\boldsymbol{\Lambda}_t^{-1} =&(\boldsymbol{\Lambda}_0 + t\boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1}
\end{aligned}$$

The last equation is the posterior variance after t periods of data. Theorem 1 implies that the posterior mean after t periods of data is

$$\begin{aligned}
\boldsymbol{\mu}_t =&\boldsymbol{\Sigma}_{t-1}(\boldsymbol{A}^T\boldsymbol{L}\vec{y}_t + \boldsymbol{\Lambda}_{t-1}\boldsymbol{\mu}_{t-1}) \\
\boldsymbol{\mu}_t =&\boldsymbol{\Lambda}_t^{-1}(\boldsymbol{A}^T\boldsymbol{L}\vec{y}_t + \boldsymbol{\Lambda}_{t-1}\boldsymbol{\mu}_{t-1})
\end{aligned}$$

In particular,

$$
\begin{aligned}
\boldsymbol{\mu}_1 =& \boldsymbol{\Lambda}_1^{-1}\boldsymbol{A}^T\boldsymbol{L}\vec{y}_1 + \boldsymbol{\Lambda}_1^{-1}\boldsymbol{\Lambda}_0\boldsymbol{\mu}_0 \\
\boldsymbol{\mu}_2 =& \boldsymbol{\Lambda}_2^{-1}\boldsymbol{A}^T\boldsymbol{L}\vec{y}_2 + \boldsymbol{\Lambda}_2^{-1}\boldsymbol{\Lambda}_1\boldsymbol{\mu}_1 \\
=& \boldsymbol{\Lambda}_2^{-1}\boldsymbol{A}^T\boldsymbol{L}\vec{y}_2 + \boldsymbol{\Lambda}_2^{-1}\boldsymbol{\Lambda}_1(\boldsymbol{\Lambda}_1^{-1}\boldsymbol{A}^T\boldsymbol{L}\vec{y}_1 + \boldsymbol{\Lambda}_1^{-1}\boldsymbol{\Lambda}_0\boldsymbol{\mu}_0) \\
=& \boldsymbol{\Lambda}_2^{-1}\boldsymbol{A}^T\boldsymbol{L}\vec{y}_2 + \boldsymbol{\Lambda}_2^{-1}\boldsymbol{A}^T\boldsymbol{L}\vec{y}_1 + \boldsymbol{\Lambda}_2^{-1}\boldsymbol{\Lambda}_0\boldsymbol{\mu}_0 \\
=& \boldsymbol{\Lambda}_2^{-1}\boldsymbol{A}^T\boldsymbol{L}\sum_{i=1}^{2}\vec{y}_i + \boldsymbol{\Lambda}_2^{-1}\boldsymbol{\Lambda}_0\boldsymbol{\mu}_0 \\
=& \boldsymbol{\Lambda}_2^{-1}(\boldsymbol{A}^T\boldsymbol{L}\sum_{i=1}^{2}\vec{y}_i + \boldsymbol{\Lambda}_0\boldsymbol{\mu}_0) \\
&\vdots \\
\boldsymbol{\mu}_t =& \boldsymbol{\Lambda}_t^{-1}(\boldsymbol{A}^T\boldsymbol{L}\sum_{i=1}^{t}\vec{y}_t + \boldsymbol{\Lambda}_0\boldsymbol{\mu}_0) \\
=& (\boldsymbol{\Lambda}_0 + t\boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1}(\boldsymbol{A}^T\boldsymbol{L}\sum_{i=1}^{t}\vec{y}_t + \boldsymbol{\Lambda}_0\boldsymbol{\mu}_0) \\
=& (\boldsymbol{\Lambda}_0 + t\boldsymbol{A}^T\boldsymbol{L}\boldsymbol{A})^{-1}(t\boldsymbol{A}^T\boldsymbol{L}\bar{\vec{y}} + \boldsymbol{\Lambda}_0\boldsymbol{\mu}_0)
\end{aligned}
$$

□

vii One Public Signal and idiosyncratic information

Inflation Expectations

Households' objective is to learn $E[\pi]$. The inflation expectations in period t for period $t+1$ are given by the expected inflation conditional on the information sets available at t. However, in order to learn the underlying mean of the inflation dynamics, households need to learn the expected bias from public statistics and from their own information on prices. By successively applying Theorem 1 a Bayesian learner household form inflation expectations in each period t given some initial priors at $t=0$. The vector of targets for household i is represented by $\vec{x}^i = (\pi, b^g, b^i)'$ for each household type i. The mean of the prior distribution over $\vec{x}^i$ is given by $\boldsymbol{\mu}^i = (\pi_0, b_0^g, b_0^i)'$ and the covariance matrix by

$$\boldsymbol{\Lambda}^{-1,i} = \begin{bmatrix} \sigma^2_{\pi,0} & 0 & 0 \\ 0 & \sigma^2_{b^g,0} & 0 \\ 0 & 0 & \sigma^2_{b^i,0} \end{bmatrix}. \tag{23}$$

when it is assumed that the priors for the inflation rate and the signals' bias are orthogonal.

The vector of signals is represented by $\vec{y}^i = (\pi^g, \pi^i)$ and the conditional distribution $p(\vec{y}|\vec{x})$ follows a Normal with mean $\boldsymbol{A}\vec{x}^i + \vec{b} = (\pi + b^g, \pi + b^i)'$, where $\vec{b} = 0$ and

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

The known conditional covariance matrix is

$$\boldsymbol{L}^{-1,i} = \begin{bmatrix} \sigma^2_g & 0 \\ 0 & \sigma^2_i \end{bmatrix}. \tag{24}$$

Note that both assumptions of Theorem 1 hold. Prior distribution is assumed to be normally distributed and the data vector $\vec{y}$ is an affine transformation of a multivariate Normal random variable[41]. Theorem 1 implies

$$E^i \begin{pmatrix} \pi_1 \\ b_1^g \\ b_1^i \end{pmatrix} = \boldsymbol{\Sigma}^i \{ \boldsymbol{A}^T \boldsymbol{L}^i (\vec{y}^i - \vec{b}) + \boldsymbol{\Lambda}^i \boldsymbol{\mu}^i \} \tag{25}$$

Solving the RHS for each component of the LHS, leads to

$$\begin{aligned} E^i(\pi_1) = \frac{1}{\Omega} \Bigg\{ & \pi_0 (\sigma^2_g + \sigma^2_{b^g,0})(\sigma^2_i + \sigma^2_{b^i,0}) \\ & + \frac{\pi^g}{\sigma^2_g} \left[\sigma^2_{\pi,0}(\sigma^2_g + \sigma^2_{b^g,0})(\sigma^2_i + \sigma^2_{b^i,0}) - \sigma^2_{\pi,0}\sigma^2_{b^g,0}(\sigma^2_i + \sigma^2_{b^i,0}) \right] \\ & + \frac{\pi^i}{\sigma^2_i} \left[\sigma^2_{\pi,0}(\sigma^2_g + \sigma^2_{b^g,0})(\sigma^2_i + \sigma^2_{b^i,0}) - \sigma^2_{\pi,0}\sigma^2_{b^g,0}(\sigma^2_g + \sigma^2_{b^g,0}) \right] \\ & - \frac{b_0^g}{\sigma^2_{b^g,0}} \left[\sigma^2_{\pi,0}\sigma^2_{b^g,0}(\sigma^2_i + \sigma^2_{b^i,0}) \right] - \frac{b_0^i}{\sigma^2_{b^i,0}} \left[\sigma^2_{\pi,0}\sigma^2_{b^i,0}(\sigma^2_g + \sigma^2_{b^g,0}) \right] \Bigg\} \end{aligned}$$

[41] The assumption of Normality for all individual price changes and that each bias is Normally distributed or constant ensure this.

$$
\begin{aligned}
E^i(b_1^g) = \frac{1}{\Omega}\Bigg\{&\sigma^2_{b^g,0}(\sigma_g^2\sigma_i^2 + \sigma_g^2\sigma^2_{\pi,0} + \sigma_i^2\sigma^2_{\pi,0} + \sigma_g^2\sigma^2_{b^i,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0})\left[\frac{b_0^g}{\sigma^2_{b^g,0}} + \frac{\pi^g}{\sigma_g^2}\right] \\
&+ \sigma^2_{\pi,0}\sigma^2_{b^g,0}\sigma^2_{b^i,0}\left[\frac{b_0^i}{\sigma^2_{b^i,0}} + \frac{\pi^i}{\sigma_i^2}\right] - \sigma^2_{\pi,0}\sigma^2_{b^g,0}(\sigma_i^2 + \sigma^2_{b^i,0})\left[\frac{\pi^g}{\sigma_g^2} + \frac{\pi^i}{\sigma_i^2} + \frac{\pi_0}{\sigma^2_{\pi,0}}\right]\Bigg\} \\
E^i(b_1^i) = \frac{1}{\Omega}\Bigg\{&\sigma^2_{b^i,0}(\sigma_g^2\sigma_i^2 + \sigma_g^2\sigma^2_{\pi,0} + \sigma_i^2\sigma^2_{\pi,0} + \sigma_i^2\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0})\left[\frac{b_0^i}{\sigma^2_{b^i,0}} + \frac{\pi^i}{\sigma_i^2}\right] \\
&+ \sigma^2_{\pi,0}\sigma^2_{b^g,0}\sigma^2_{b^i,0}\left[\frac{b_0^g}{\sigma^2_{b^g,0}} + \frac{\pi^g}{\sigma_g^2}\right] - \sigma^2_{\pi,0}\sigma^2_{b^i,0}(\sigma_g^2 + \sigma^2_{b^g,0})\left[\frac{\pi^g}{\sigma_g^2} + \frac{\pi^i}{\sigma_i^2} + \frac{\pi_0}{\sigma^2_{\pi,0}}\right]\Bigg\}
\end{aligned}
$$

where

$$
\Omega \equiv \sigma_g^2\sigma_i^2 + \sigma_g^2\sigma^2_{\pi,0} + \sigma_i^2\sigma^2_{\pi,0} + \sigma_g^2\sigma^2_{b^i,0} + \sigma_i^2\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0} + \sigma^2_{b^g,0}\sigma^2_{b^i,0}
$$

By rearranging and simplifying terms

$$
\begin{aligned}
E^i(\pi_1) = \pi_0&\left(\frac{\sigma_g^2\sigma_i^2 + \sigma_g^2\sigma^2_{b^i,0} + \sigma_i^2\sigma^2_{b^g,0} + \sigma^2_{b^g,0}\sigma^2_{b^i,0}}{\Omega}\right) \\
&+ (\pi^g - b_0^g)\left(\frac{\sigma^2_{\pi,0}\sigma_i^2 + \sigma^2_{\pi,0}\sigma^2_{b^i,0}}{\Omega}\right) + (\pi^i - b_0^i)\left(\frac{\sigma^2_{\pi,0}\sigma_g^2 + \sigma^2_{\pi,0}\sigma^2_{b^g,0}}{\Omega}\right) \\
E^i(b_1^g) = b_0^g&\left(\frac{\sigma_g^2\sigma_i^2 + \sigma_g^2\sigma^2_{\pi,0} + \sigma_i^2\sigma^2_{\pi,0} + \sigma_g^2\sigma^2_{b^i,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0}}{\Omega}\right) \\
&+ (\pi^g - \pi_0)\left[\frac{\sigma^2_{b^g,0}(\sigma_i^2 + \sigma^2_{b^i,0})}{\Omega}\right] + [\pi^g - (\pi^i - b_0^i)]\left(\frac{\sigma^2_{\pi,0}\sigma^2_{b^g,0}}{\Omega}\right) \\
E^i(b_1^i) = b_0^i&\left(\frac{\sigma_g^2\sigma_i^2 + \sigma_g^2\sigma^2_{\pi,0} + \sigma_i^2\sigma^2_{\pi,0} + \sigma_i^2\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0}}{\Omega}\right) \\
&+ (\pi^i - \pi_0)\left[\frac{\sigma^2_{b^i,0}(\sigma_g^2 + \sigma^2_{b^g,0})}{\Omega}\right] + [\pi^i - (\pi^g - b_0^g)]\left(\frac{\sigma^2_{\pi,0}\sigma^2_{b^i,0}}{\Omega}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
E^i(\pi_1) &= \pi_0(1 - \psi_1 - \psi_2) + (\pi^g - b_0^g)\psi_1 + (\pi^i - b_0^i)\psi_2 \\
E^i(b_1^g) &= b_0^g(1 - \phi_1 - \phi_2) + (\pi^g - \pi_0)\phi_1 + [\pi^g - (\pi^i - b_0^i)]\phi_2 \\
E^i(b_1^i) &= b_0^i(1 - \omega_1 - \omega_2) + (\pi^i - \pi_0)\omega_1 + [\pi^i - (\pi^g - b_0^g)]\omega_2
\end{aligned} \tag{26}
$$

where

$$\psi_1 \equiv \frac{\sigma^2_{\pi,0}\sigma^2_i + \sigma^2_{\pi,0}\sigma^2_{b^i,0}}{\sigma^2_g\sigma^2_i + \sigma^2_g\sigma^2_{\pi,0} + \sigma^2_i\sigma^2_{\pi,0} + \sigma^2_g\sigma^2_{b^i,0} + \sigma^2_i\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0} + \sigma^2_{b^g,0}\sigma^2_{b^i,0}}$$
$$\psi_2 \equiv \frac{\sigma^2_{\pi,0}\sigma^2_g + \sigma^2_{\pi,0}\sigma^2_{b^g,0}}{\sigma^2_g\sigma^2_i + \sigma^2_g\sigma^2_{\pi,0} + \sigma^2_i\sigma^2_{\pi,0} + \sigma^2_g\sigma^2_{b^i,0} + \sigma^2_i\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0} + \sigma^2_{b^g,0}\sigma^2_{b^i,0}}$$
$$\phi_1 \equiv \frac{\sigma^2_{b^g,0}\sigma^2_i + \sigma^2_{b^g,0}\sigma^2_{b^i,0}}{\sigma^2_g\sigma^2_i + \sigma^2_g\sigma^2_{\pi,0} + \sigma^2_i\sigma^2_{\pi,0} + \sigma^2_g\sigma^2_{b^i,0} + \sigma^2_i\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0} + \sigma^2_{b^g,0}\sigma^2_{b^i,0}}$$
$$\phi_2 \equiv \frac{\sigma^2_{\pi,0}\sigma^2_{b^g,0}}{\sigma^2_g\sigma^2_i + \sigma^2_g\sigma^2_{\pi,0} + \sigma^2_i\sigma^2_{\pi,0} + \sigma^2_g\sigma^2_{b^i,0} + \sigma^2_i\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0} + \sigma^2_{b^g,0}\sigma^2_{b^i,0}}$$
$$\omega_1 \equiv \frac{\sigma^2_{b^i,0}\sigma^2_g + \sigma^2_{b^i,0}\sigma^2_{b^g,0}}{\sigma^2_g\sigma^2_i + \sigma^2_g\sigma^2_{\pi,0} + \sigma^2_i\sigma^2_{\pi,0} + \sigma^2_g\sigma^2_{b^i,0} + \sigma^2_i\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0} + \sigma^2_{b^g,0}\sigma^2_{b^i,0}}$$
$$\omega_2 \equiv \frac{\sigma^2_{\pi,0}\sigma^2_{b^i,0}}{\sigma^2_g\sigma^2_i + \sigma^2_g\sigma^2_{\pi,0} + \sigma^2_i\sigma^2_{\pi,0} + \sigma^2_g\sigma^2_{b^i,0} + \sigma^2_i\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^g,0} + \sigma^2_{\pi,0}\sigma^2_{b^i,0} + \sigma^2_{b^g,0}\sigma^2_{b^i,0}}$$

For any given period t, equations in (26) can be expressed as[42]

$$\begin{aligned}
E[\pi^{e,i}_{t+1|t}|\mathcal{I}^i_t] &= \pi^{e,i}_{t|t-1}(1-\psi_1-\psi_2) + (\pi^g_{t|t} - b^g_{t|t-1})\psi_1 + (\pi^i_{t|t} - b^i_{t|t-1})\psi_2 \\
E[b^g_{t+1|t}|\mathcal{I}^i_t] &= b^g_{t|t-1}(1-\phi_1-\phi_2) + (\pi^g_{t|t} - \pi^{e,i}_{t|t-1})\phi_1 + [\pi^g_{t|t} - (\pi^i_{t|t} - b^i_{t|t-1})]\phi_2 \\
E[b^i_{t+1|t}|\mathcal{I}^i_t] &= b^i_{t|t-1}(1-\omega_1-\omega_2) + (\pi^i_{t|t} - \pi^{e,i}_{t|t-1})\omega_1 + [\pi^i_{t|t} - (\pi^g_{t|t} - b^g_{t|t-1})]\omega_2
\end{aligned} \tag{27}$$

viii Two Public Signals and idiosyncratic information

Inflation Expectations

$$\vec{x} = \begin{bmatrix}\pi \\ b^g \\ b^i\end{bmatrix},\ \mu = \begin{bmatrix}\pi_0 \\ b^g_0 \\ b^i_0\end{bmatrix},\ \vec{y} = \begin{bmatrix}\pi^g \\ \pi^i \\ \pi^u\end{bmatrix},\ \Lambda^{-1} = \begin{bmatrix}\sigma^2_{\pi,0} & 0 & 0 \\ 0 & \sigma^2_{b^g,0} & 0 \\ 0 & 0 & \sigma^2_{b^i,0}\end{bmatrix},\ \mathcal{A} = \begin{bmatrix}1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0\end{bmatrix},$$
$$\vec{b} = 0,\ L^{-1} = \begin{bmatrix}\sigma^2_g & 0 & 0 \\ 0 & \sigma^2_i & 0 \\ 0 & 0 & \sigma^2_u\end{bmatrix},\ A\vec{x} + \vec{b} = \begin{bmatrix}\pi + b^g \\ \pi + b^i \\ \pi\end{bmatrix}.$$

[42] These expressions follow from assuming orthogonal priors. See the section that describe the recursive application to see the evolution of Bayesian beliefs.

The signals are given by

$$\begin{aligned} \pi^g &= \bar{\pi}^g + b^g, \quad \bar{\pi}^g \sim (\mu^g,\, \sigma_g^2) \\ \pi^i &= \bar{\pi}^i + b^i, \quad \bar{\pi}^i \sim (\mu^i,\, \sigma_i^2) \\ \pi^u &= \bar{\pi}^u, \quad \bar{\pi}^u \sim (\mu^u,\, \sigma_u^2) \end{aligned} \tag{28}$$

Posterior beliefs

$$\begin{aligned} E(\pi_1) &= \pi_0(1-\psi_1-\psi_2-\psi_3) + (\pi^g - b_0^g)\psi_1 + (\pi^i - b_0^i)\psi_2 + \pi^u\psi_3 \\ E(b_1^g) &= b_0^g(1-\phi_1-\phi_2-\phi_3) + (\pi^g - \pi_0)\phi_1 + (\pi^g - (\pi^i - b_0^i))\phi_2 + (\pi^g - \pi^u)\phi_3 \\ E(b_1^i) &= b_0^i(1-\omega_1-\omega_2-\omega_3) + (\pi^i - \pi_0)\omega_1 + (\pi^i - (\pi^g - b_0^g))\omega_2 + (\pi^i - \pi^u)\omega_3 \end{aligned} \tag{29}$$

Appendix C: Additional simulation results